

COMPARISON OF INTERPOLATION AND CONTOUR  
CONSTRUCTION METHODS APPLICABLE TO THE  
MX VALLEY SOIL SAMPLING PROJECT

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Haluk Ünaldi



# NAVAL POSTGRADUATE SCHOOL

## Monterey, California



# THESIS

COMPARISON OF INTERPOLATION AND  
CONTOUR CONSTRUCTION METHODS  
APPLICABLE TO THE MX VALLEY  
SOIL SAMPLING PROJECT

by

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Comparison of Interpolation and Contour  
Construction Methods Applicable to the MX Valley  
Soil Sampling Project

by

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## ABSTRACT

The objective is to obtain and compare interpolation methods and contour plot techniques, based on data from sites whose locations are irregular or scattered. Data from the 1980 Fugro report [Ref. 1] on MX valley soil samples serves as a test bed.

Two groups of data, seismic p-wave velocities and surface soil depth were studied using six different interpolation methods and three different contour plot techniques. Comparison of the interpolation methods and of the contour plot techniques are made.



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## I. INTRODUCTION

Detailed investigations of seismic p-wave velocities and depth of the surface soil layer were performed in the Ralston Valley, Nevada, by Fugro, Inc. [Ref. 1], for the U.S. Army Engineer Waterways Experiment Station (W.E.S.) as part of the preliminary work on the MX missile system deployment. Sixteen locations were chosen in the Ralston Valley by W.E.S. for their study organized into four sets of sites (coded RA, RB, RC, RD) located in each of the four predominant surficial soil types (coded 5I, 5Y, U, 4U), and an additional site marked RU2 identified in the Valley.

The data source for identified sites [Ref. 2] gives the (X,Y) coordinates (one mile to the inch) of the boring locations relative to the map's lower left grid mark, the seismic p-wave velocities (MPS - meters per second) of the surface layer, and the depth of that layer (meters) for the 2B line; see Appendix A. The site identifiers and locations for the Ralston Valley sampling plan can be found in Appendix B. The immediate concern is to develop contours describing the depth of the top layer, and of the compression wave speed in the top layer, in the entire Valley.

Based on these limited data, the mathematical problem is that of constructing a smooth bivariate function  $F(X,Y)$  which takes on specified values  $F(X_k, Y_k) = F_k$  for  $K = 1, \dots, N$ .



Franke [Ref. 3,4,5,6] has made a rather extensive general study of the modern interpolation methods. Most of the methods require the user to specify a parameter (or several), but these parameters can be determined by computational experiment [Ref. 4]. The interpolation methods selected for specific inclusion in the current work are a subset of these methods, see Table 1. Furthermore, contour plotting, specifically non-IMSL subroutine CONTUR, subroutine CONISD and subroutine PLT3D1, were exercised for comparison. Basic FORTRAN programs for the first five interpolation methods were obtained from Professor R. Franke [Ref. 3,4,5,6].

The various interpolation methods were used to supply function values at a gridwork of reference points. In order to compare the methods, the mean and standard deviation of the values were produced at each grid point. Also it is important to identify which methods generates sharply different interpolation values compared to the other methods. For this purpose, the same FORTRAN program generates the absolute differences from the mean values for each method.

FORTRAN programs for all of the methods can be obtained by writing the advisor.

In the next chapter we will describe a number of interpolation methods. Chapter 3 describes employment of the Ralston Valley data as a way of comparing these methods. Chapter 4 discusses contour plot techniques, and the conclusions from using these procedures are given in Chapter 5. Graphical results are presented in a series of appendices.





TABLE 1

## INTERPOLATION METHODS

- 1) Inverse Distance Weighted Methods
  - A) Shepard's Method
  - B) Modified Shepard's Method
  - C) Modified Linear Shepard's Method
  - D) Modified Shepard's Method Boolean Sum Plane
  - E) Modified McLain Method
  - F) Quadratic Shepard's Method
  - \*G) Modified Quadratic Shepard's Method
  - H) McLain Method
- 2) Franke's Methods (Rectangle based blending methods)
  - A) Franke's Method (Mode One)
  - B) Franke's Method (Mode Three)
  - \*C) Franke's Method (Thin plate local functions)
- 3) Triangle Based Blending Methods
  - A) Nielson-Franke Linear Triangle Method
  - \*B) Nielson-Franke Quadratic Triangle Method
- 4) Finite Element Based Methods
  - A) Akima's Method
  - B) Akima's Method - Modification Two
  - C) Akima's Method - Modification One
  - D) Akima's Method - Modification Three
  - \*E) Nielson's Minimum Norm Network
  - F) Lawson's Method
- 5) Foley's Method
  - A) Generalized Newton Interpolant
  - B) TF Delta Sum Bernstein Interpolant
  - C) Iterated Delta Sums: TF Delta Sum Bicubic Spline
  - \*D) Iterated Delta Sums: A Shepard's Method Delta Sum Bic. Sp.
- 6) Nodal Basis Function Type Methods
  - A) Rotated Gaussian
  - \*B) Hardy's Multiquadric
  - C) Hardy's Reciprocal Multiquadric
  - D) Duchon's Radial Cubic Method
  - E) Duchon's Thin Plate Function
  - F) Rotated B-Spline

\* Methods chosen for comparison study.



## II. INTERPOLATION METHODS

The problem being addressed here is that of constructing a smooth bivariate function,  $F(X,Y)$ , which takes on specified values  $F(X_k, Y_k) = F_k$ ,  $k=1, \dots, N$  based on point data from scattered, irregular locations. The bivariate function,  $F(X,Y)$  is smooth, (i.e., it has at least continuous first partial derivatives), and the points  $(X_k, Y_k)$  are 'scattered', that is, not necessarily conforming to some regular pattern.

Some interpolation methods, called 'global', are sensitive to all the data points: the addition or deletion of a data point, or a change of one of the coordinates of a data point, will propagate throughout the entire domain of definition. Other interpolation methods are called 'local', that is, insensitive to changes in the data points which are sufficiently distant from the location at hand.

Franke, R. [Ref. 3,4,5,6] has made an extensive general study of new interpolation methods. He produced a classification of the interpolation methods [Ref. 4], as shown in Table 1. Under his six main types, there are a total of 29 different methods. All the methods appearing in the same group are either local or global and use the same basic idea; several are modifications of the previously described ones.

Six methods, one from each of the six main types, are described below.



## A. MODIFIED QUADRATIC SHEPARD'S METHOD

The modified quadratic Shepard's method was derived from the inverse distance weighted method. All methods of this type which are considered may be viewed as generalizations of Shepard's method. The basic Shepard's method is

$$F(X,Y) = \frac{\sum_{k=1}^N W_k(X,Y) \cdot Q_k(X,Y)}{\sum_{k=1}^N W_k(X,Y)}$$

where  $W_k(X,Y) = d_k^{-\mu}$ . Typically  $\mu = 2$ , although other values may be used. Here  $d_k^2 = ((X-K_k)^2 + (Y-Y_k)^2)$ .

In the modified quadratic Shepard's method the weights for obtaining the nodal functions (quadratics) are taken as

$$W_k(X,Y) = \left[ \frac{(R_q - d_k)}{(R_q + d_k)} \right]^2 \quad \text{and} \quad R_q = 1/2 \sqrt{N_q/N} \cdot D$$

where  $D$  is the diameter of the point set, and a nominal value for  $N_q$ , determined by computational experiment [Ref. 4], is  $N_q = 18$ . The diameter of the point set is defined

$$D^2 = \max_{I,J} \left[ (X_i - X_j)^2 + (Y_i - Y_j)^2 \right]$$

The nodal functions  $Q_k(X,Y)$  are defined as





$$Q_k(X,Y) = F_k + \overline{a_{k2}}(X-X_k) + \overline{a_{k3}}(Y-Y_k) + \overline{a_{k4}}(X-X_k)^2 \\ + \overline{a_{k5}}(X-X_k) \cdot (Y-Y_k) + \overline{a_{k6}}(Y-Y_k)^2.$$

and for the coefficients one must solve the least squares problem,

$$\begin{aligned} \text{MIN} \quad & \sum_{i=1}^N W_k \left[ F_k + a_{k2}(X_i - X_k) + a_{k3}(Y_i - Y_k) \right. \\ & + a_{k4}(X_i - X_k)^2 + a_{k5}(X_i - X_k) \cdot (Y_i - Y_k) \\ & \left. + a_{k6}(Y_i - Y_k)^2 - F_i \right]^2. \end{aligned}$$

$a_{kj}, j=2, \dots, 6 \quad i \neq k$

Complete details are given in [Ref. 4,5].

## B. LOCAL THIN PLATE SPLINES METHOD

This is known as Franke's method [Ref. 4]. The idea is to represent the interpolation function as

$$F(X,Y) = \sum_{i,j} W_{ij}(X,Y) \cdot Q_{ij}(X,Y)$$

where the  $W_{ij}(X,Y)$  are local weight functions [Ref. 6]. Here we choose the  $W_{ij}$  so that  $W_{ij} = 1$ , and the local approximations  $Q_{ij}(X,Y)$  interpolate at points where the corresponding weight function is non-zero. The weight functions are products of piecewise hermite cubics [Ref. 6].



The local approximations for this method are taken to be the thin plate splines and have the form

$$Q_{ij}(X,Y) = \sum_{k \in I} A_k \cdot d_k^2 \cdot \text{LOG } d_k + a + b \cdot X + c \cdot Y$$

where  $d_k^2 = ((X-X_k)^2 + (Y-Y_k)^2)$  .

And I is the set of indices k for which  $(X_k, Y_k, F_k)$  is a point to be interpolated by  $Q_{ij}(X,Y)$  , and is defined by

$$I = \left\{ K : Q \text{ is to take the value } F_k \text{ at } (X_k, Y_k) \right\} .$$

The coefficients  $A_k$  , and a , b and c are determined by a certain linear system of equations , (see [Ref. 6]) .

#### C. NIELSON-FRANKE QUADRATIC TRIANGLE METHOD

All methods listed under triangle based blending methods [Ref. 4] are conceptually similar to the inverse distance weighted methods. The interpolation function for the quadratic triangle method is

$$F(X,Y) = W_i(X,Y) \cdot Q_i(X,Y) + W_j(X,Y) \cdot Q_j(X,Y) + W_k(X,Y) \cdot Q_k(X,Y)$$

The first step in the present method is to partition the plane into triangles by connecting neighboring data points based upon the min-max angle criterion as described by Franke in [Ref. 5]. The quadratic triangle method uses the inverse distance weighted quadratic  $Q_i(X,Y)$  which is also used in the modified quadratic Shepard's method, see section 2.A.



A significant difference from the inverse distance weighted method is that the weight functions for the triangle based blending method are based on a triangulation of the convex hull of the point set  $\{ (X_k, Y_k) \}$ . The method is not sensitive to the triangulation technique. If a triangulation is in existence for other purposes, it can be used.

In short, the method is to define the nodal functions  $Q_k, k=1, \dots, N$ , as in the modified quadratic Shepard's method, form a triangulation of the points, determine the weight functions [Ref. 5] and compute interpolant  $F(X, Y)$ .

#### D. NIELSON'S MINIMUM NORM NETWORK METHOD

Nielson's minimum norm network method [Ref. 4,5], can be found under the finite element based methods in Table 1. Like the Nielson-Franke quadratic triangle method, this global method is based upon a triangulation.

The method consists of three separate steps; triangulation curve network, and blending [Ref. 7]. The points  $(X_k, Y_k), k=1, \dots, N$  are used as the vertices of a triangulation of the convex hull of these points. Alternatively, if a triangulation already exists, it can be used. The curve network step involves the solution of a certain minimum norm problem and requires first partial derivatives in its discretized form. These are obtained by assuming a cubic variation along each edge in the triangulation and minimizing the integral of the second derivative squared [Ref. 7].



This method does not provide extrapolation outside the convex hull, but the triangular blending method can be used to extend the curve network to the entire hull. In a triangular exterior region, the function is taken to be the linear function determined by the value and slopes at the vertex [Ref. 7].

#### E. GENERALIZED NEWTON DELTA SUM BICUBIC SPLINE METHOD

Foley's methods [Ref. 4] involve several ideas. The interpolant is taken as either the generalized Newton interpolant, or a form of Shepard's method. The use of generalized Newton type interpolant is involved in them prominently, because the best performance for various test functions is provided by the iterated delta sum methods using the generalized Newton interpolant with natural bicubic splines.

The generalized Newton interpolant takes the form

$$T_N(X,Y) = \sum_{k=1}^N a_k \cdot W_k(X,Y) ,$$

where

$$a_k = \frac{f_k - T_{k-1}(x_k, y_k)}{W_k(X_k, Y_k)}$$

and  $W_k(X,Y)$  has the property  $W_k(X_i, Y_i) = 0 \quad i=1,2,\dots,k-1$ .

This function is dependent upon the order of the points, and so Foley's scheme involves an ordering process [Ref. 4].

After constructing a bicubic spline interpolant for the grid points, one adds a generalized Newton interpolant for the difference between data and the bicubic splines, obtaining





an interpolant. This process is termed a 'delta sum' by Foley [Ref. 4].

#### F. HARDY'S MULTIQUADRIC METHOD

Hardy's multiquadric method [Ref. 8,9] can be thought of as being under the global basis function type. A multiquadric surface can be represented by

$$\sum_{j=1}^N C_j \left[ (X_j - X_i)^2 + (Y_j - Y_i)^2 + C \right]^{1/2} = Z_i, \quad i=1, \dots, N.$$

The given problem data provide a set of cartesian coordinates on the surface ranging from  $X_1, Y_1, Z_1$  to  $X_N, Y_N, Z_N$ ; the quadratic term coefficients  $C_1$  to  $C_N$  are unknown. The coordinates of the  $N$  data points can be substituted into the definition of the surface and only the coefficients  $C_j$  are left as unknown. A problem here is that of solving a system of  $N$  linear equations with  $N$  unknowns. A nominal value for  $C$  was determined by computational experiment and the value  $C = 0$  was chosen.

If a vector of unknowns is to be used, define

$$[C_j] = \begin{bmatrix} C_1 \\ C_2 \\ \cdot \\ \cdot \\ C_N \end{bmatrix}$$

then each known element becomes



$$\left[ (X_j - X_i)^2 + (Y_j - Y_i)^2 \right]^{1/2} = a_{ij} ,$$

and the matrix  $[a_{ij}] = A$ , which is (NXN) coefficient matrix.

Upon

defining which

$$[z_i] = \begin{bmatrix} z_1 \\ z_2 \\ . \\ . \\ z_N \end{bmatrix}$$

A multiquadric surface reduces to  $AxC = Z$  and, as usual, the solution is  $C = A^{-1}Z$ .

When the known constraints  $C_j$  are substituted into the multiquadric surface formula, we have the required equation of a surface, which fits the data points exactly and which provides logical interpolation at intermediate points.



### III. COMPARISON OF INTERPOLATION METHODS USING RALSTON VALLEY DATA

Contour construction for seismic p-wave velocities and depth data can be seen in Appendix C and in Appendix F, respectively, for the various interpolation methods and with the three different contour plot techniques. Each method appears in the order given in Table 1. Each method is displayed using first the subroutine CONTUR, then the subroutine CONISD, and lastly the subroutine PLT3D1. Subroutine CONTUR cannot outline the Ralston Valley, but gives contour constructions in rectangular form with the x-axis ranging from one to seventeen miles and the y-axis from one to twenty five miles. The subroutine CONISD can outline the Ralston Valley with boundary lines. All three dimensional pictures were generated by subroutine PLT3D1.

Appendix D shows the mean and standard deviation of the various contour constructions for the seismic p-wave velocities. Numerical values for the mean and standard deviation at each grid point come from six different interpolation methods. Appendix G is similar to Appendix D, but it is for the depth of the surface soil layer. Convex contour lines are formed for the standard deviations values as can be seen in Appendices D and G. Small standard deviation values appear in data point region, and become larger as one moves



toward the edges. In other words, more accurate prediction occurs in data point region and less accurate prediction outside this region.

Deviations from mean values for all interpolation methods can be seen in Appendix E for seismic p-wave velocities and in Appendix H for depth values. The 1, 10, 50 and 100 contours show the absolute differences between the given method and mean values for seismic p-wave velocities. The 0.1, 0.5, 1.0 contours for depth data display quite convex regions for all methods. Clearly, it can be seen that; there are no significant differences between methods in terms of deviations from mean values for grid points especially in data region. This result shows that in the data points region all methods work with almost same accuracy and outside the region there are too great differences. There is no way to compare them outside the region.





#### IV. CONTOUR PLOT TECHNIQUES

The interpolation values provided by any of the methods can be seen in the computer outputs, but their interpretation is highly difficult without contour construction. For interpretation purposes, computer programs for drawing plots on an off-line plotter are available at the Naval Postgraduate School computer center.

The subprograms CONTUR, CONISD and PLT3D1 serve the common purpose but have different capabilities.

All the subprograms have the same limitation in that they use a linear interpolation process to find the values of the points along contour segments. Also, there is a limit in the resolution of the off-line plotter, namely, .005 inch in both the X and Y directions for the versatec plotter.

##### A. SUBROUTINE CONTUR

Given a matrix of numerical values of  $Z = (X,Y)$ , the program CONTUR generates a contour graph on which one or more contour levels are drawn. This non-IMSL subroutine uses a two-dimensional array and turns out a two-dimensional off-line plot. Interior and exterior contour segments and labels (if requested) for exterior contour segments can be seen. Labeling of the interior segments represent local maxima.



#### Advantages:

- 1) The subroutine has a minimum number of arguments (10) compared with other plot techniques.
- 2) A rather big two-dimensional array can be used, up to 1797 data points.
- 3) It takes approximately one fiftieth of the CPU time that would be required by subroutine CONISD.

#### Disadvantages:

- 1) Irregularly spaced data cannot be contoured.
- 2) Irregular boundary capability is not available, and there is no option for a 'cut-out' area.
- 3) The width and the height of the contour graph must be specified as integers, and are not adjustable scale parameters.
- 4) The scale values (if requested) one inch apart on the exterior frame of the contour graph increase from north to south. This is not oriented properly for our use, which is a lower left grid oriented system.
- 5) Limited labeling. The x-axis and y-axis cannot be labeled.
- 6) There is no option for smoothing the contour lines.

#### B. SUBROUTINE CONISD

This non-IMSL subroutine produces a drawn contour map on an off-line plotter for given irregularly spaced data points. Each data point is a triad of X,Y and Z values where  $Z = f(X,Y)$ . There may be one or more 'cut-out areas' which are not contour.



Advantages:

- 1) Irregularly spaced data can be drawn.
- 2) Irregular boundary capability is available for one or more 'cut-out areas'.
- 3) The scale for the x-axis and y-axis are defined as real numbers, i.e., adjustable scale.
- 4) X-axis and y-axis can be labeled.
- 5) There is an option for smoothing the contour lines.

Disadvantages:

- 1) It takes much more CPU time than the other plot techniques.

C. SUBROUTINE PLT3D1

A subroutine PLT3D1 produces three-dimensional perspective or isometric plots on an off-line plotter for given a two-dimensional matrix of numerical values of  $Z = (X,Y)$ . Labeling and scaling are not available for this reason, it can be used only for general purposes.

A memorandum about this subprogram [Ref. 12] can be requested from W.R. Church computer center. This memorandum includes a listing of the computer program, the algorithm that is used by the program, and the axes orientation, rotation and projection are clarified by illustrations.

Advantages:

- 1) This routine gives a nice overview to given area by  $Z = (X,Y)$ .
- 2) The vantage point can be changed.
- 3) The scale is adjustable.



Disadvantages:

- 1) There is no option for 'cut-out areas'.
- 2) There is no labeling.





## V. CONCLUSIONS AND RECOMMENDATIONS

Seismic p-wave velocities or depth values for any grid point in Ralston Valley can be found from computer outputs or contour constructions for any particular interpolation method or for the mean of the six different methods. The standard deviations for each grid point are also available. The means and standard deviations may be useful for the comparison of methods.

The computer outputs for standard deviations in Appendix E and H show that accuracy is high in the data point region and at some undecided distance from this region; but this distance cannot be measured. In the data point region all interpolation methods gives essentially the same result, regardless of whether they are global or local.

Outside the data point region all methods generate rather different values and there is no way to decide which one is close to true values. If the point of interest is in the data point region any of the interpolation methods can be used.

According to Appendix C and F; none of the methods can find a maximum or minimum point outside the region. Table 2 gives approximate CPU times for all of the studied interpolation methods. If minimal computer time is required for a given problem, the triangle based quadratic method is suitable.



TABLE 2

## CPU TIMES FOR STUDIED INTERPOLATION METHODS

Method	CPU Times (Milli-seconds)	Local/ Global
Modified Quadratic Shepard's Method	12.9	Local
Local Thin Plate Splines Method	19.6	Global
Triangle Based Quadratics Method	8.4	Local
Hardy's Multiquadric Method	19.3	Global
Delta Sum Bicubic Spline Method	10.1	Global
Nielson's Minimum Norm Network Method	16.0	Global



## APPENDIX A







TABLE 3

DATA SELECTED FOR TEST BED


Side Designation		Coordinates		P-Wave Vel.	Depth
		X	Y	NPS	M.
1.	RA5I	9.25	30.25	561	3.0
2.	RB5I	5.69	25.5	366	1.0
3.	RC5I	4.75	20.81	279	0.9
4.	RD5I	5.75	14.75	347	1.5
5.	RA5Y	5.59	26.25	341	1.5
6.	RB5Y	11.75	24.19	366	0.8
7.	RC5Y	7.06	20.0	323	1.2
8.	RD5Y	7.31	10.31	329	0.8
9.	RAU	7.31	28.19	363	1.2
10.	RBU	9.25	19.31	335	1.6
11.	RCU	8.75	11.25	317	1.4
12.	RDU	17.44	10.94	378	1.5
13.	RA4U	10.12	23.44	341	1.5
14.	RB4U	12.25	18.56	372	1.5
15.	RC4U	11.44	14.38	372	1.5
16.	RD4U	15.44	12.50	381	0.5
17.	RU2	6.00	27.19	357	1.2



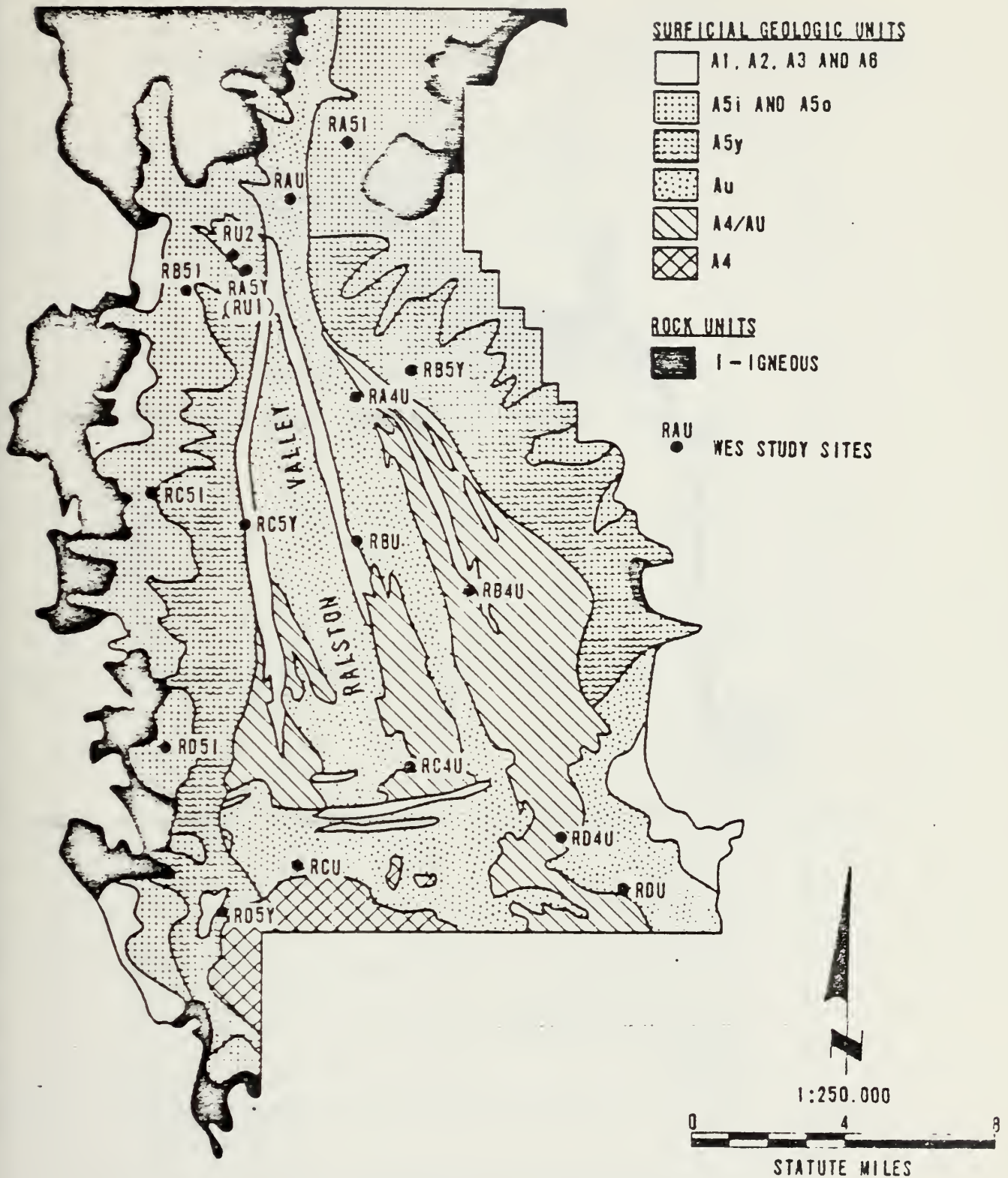
**EXPLANATION****SURFICIAL GEOLOGIC UNITS**

-  A1, A2, A3 AND A6
-  A5i AND A5o
-  A5y
-  Au
-  A4/AU
-  A4

**ROCK UNITS**

-  I - IGNEOUS

-  RAU
- WES STUDY SITES



Site numbers and locations for Ralston Valley soil sampling plan.

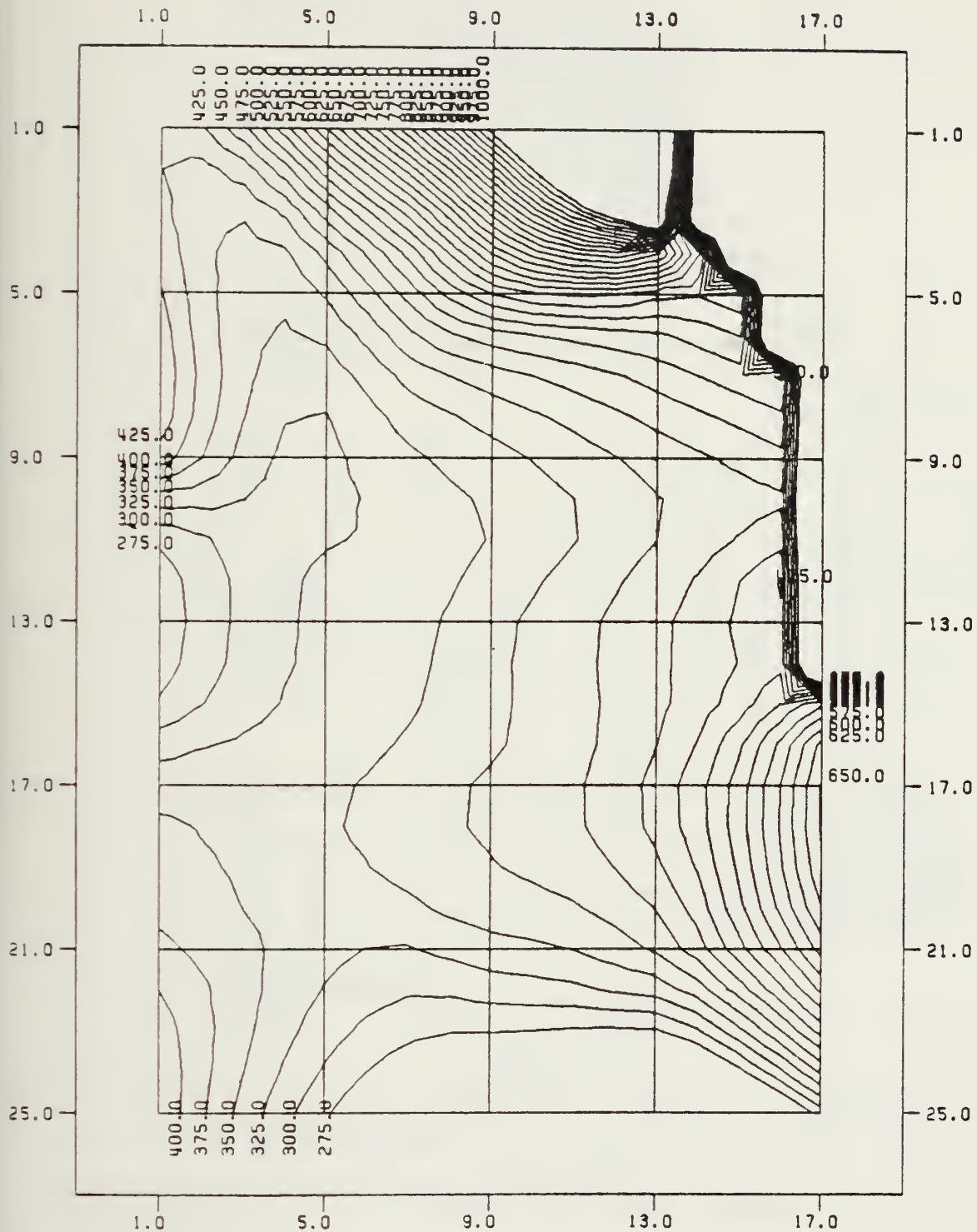
Figure 1





# P-WAVE VEL. (NPS)

## QUADRATIC SHEPARDS METHOD



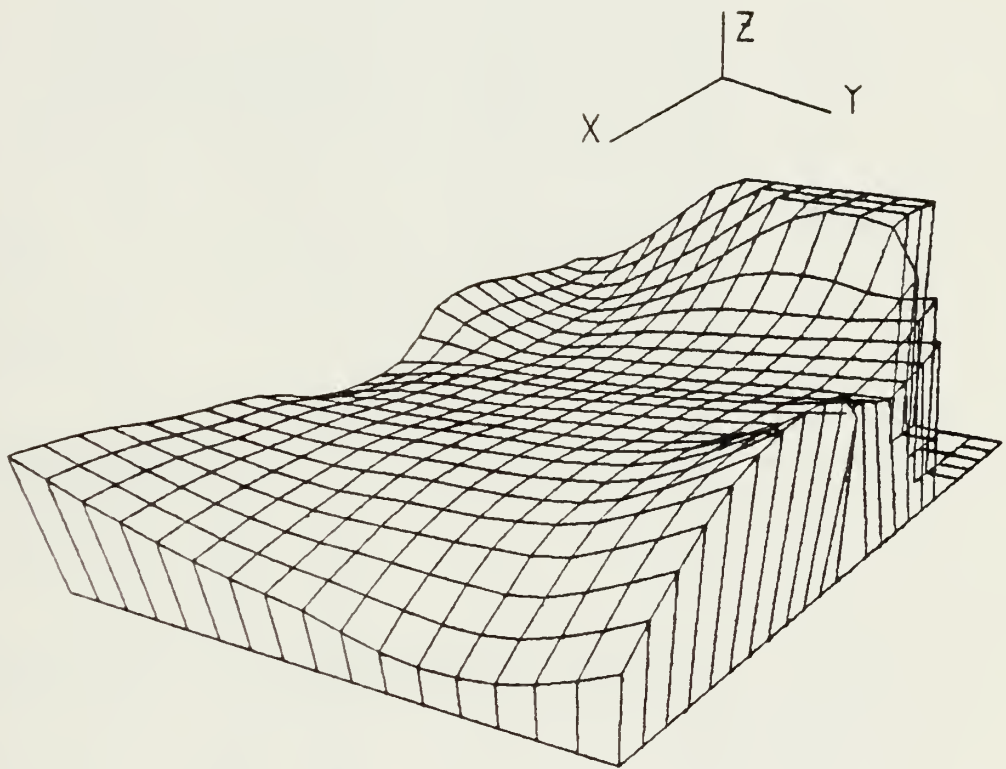
(Using subroutine CONTUR)

Figure 2







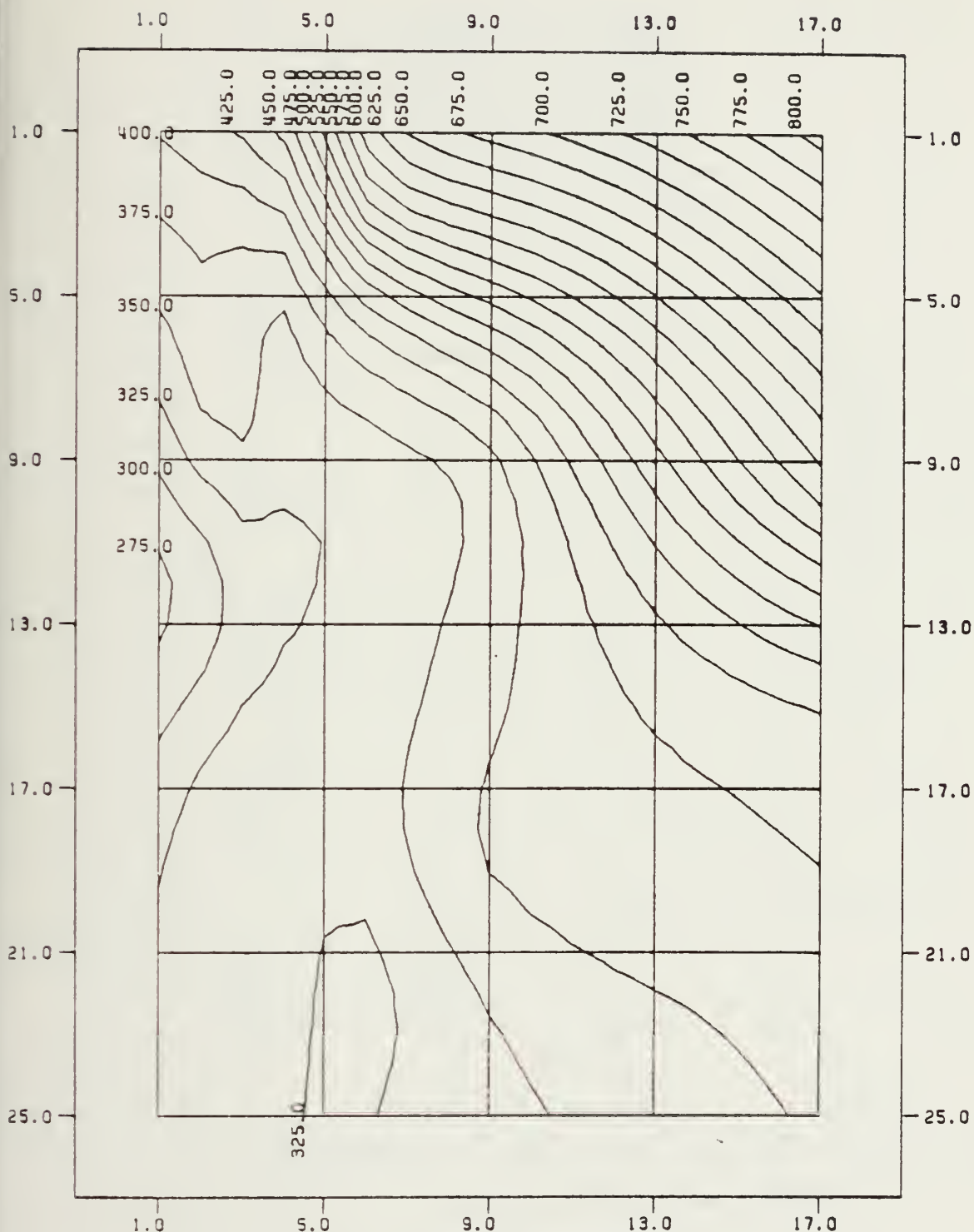


P-WAVE VEL. (NPS)  
QUADRATIC SHEPARD'S METHOD  
(Using subroutine PLT3D1)

Figure 4



# P-WAVE VEL. (NPS) LOCAL THIN PLATE SPLINE M.

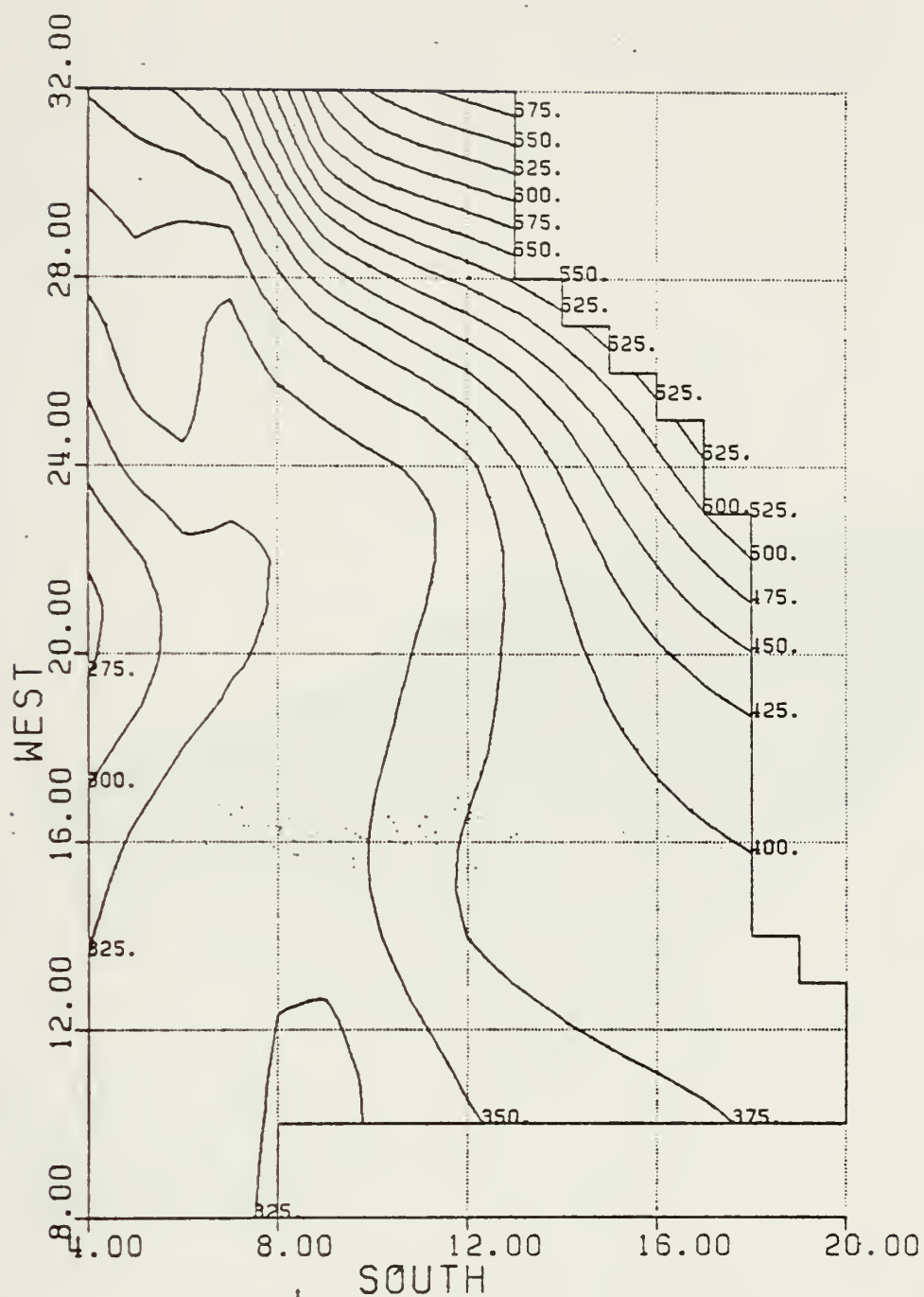


(Using subroutine CONTUR)

Figure 5



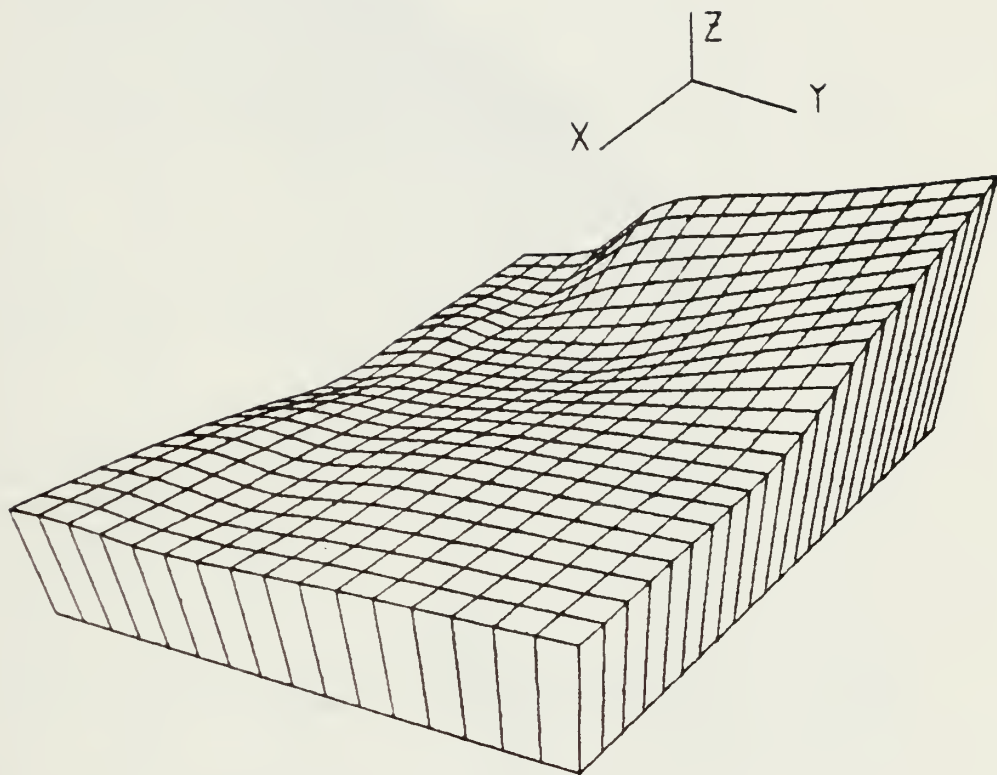




P-WAVE VEL. (NPS)  
 LOCAL THIN PLATE SPLINE M.  
 (Using subroutine CONISD)

Figure 6





P-WAVE VEL. (NPS)  
LOCAL THIN PLATE SPLINE M.  
(Using subroutine PLT3D1)

Figure 7



# P-WAVE VEL. (NPS)

## TRIANGLE BASED QUAD. METHOD

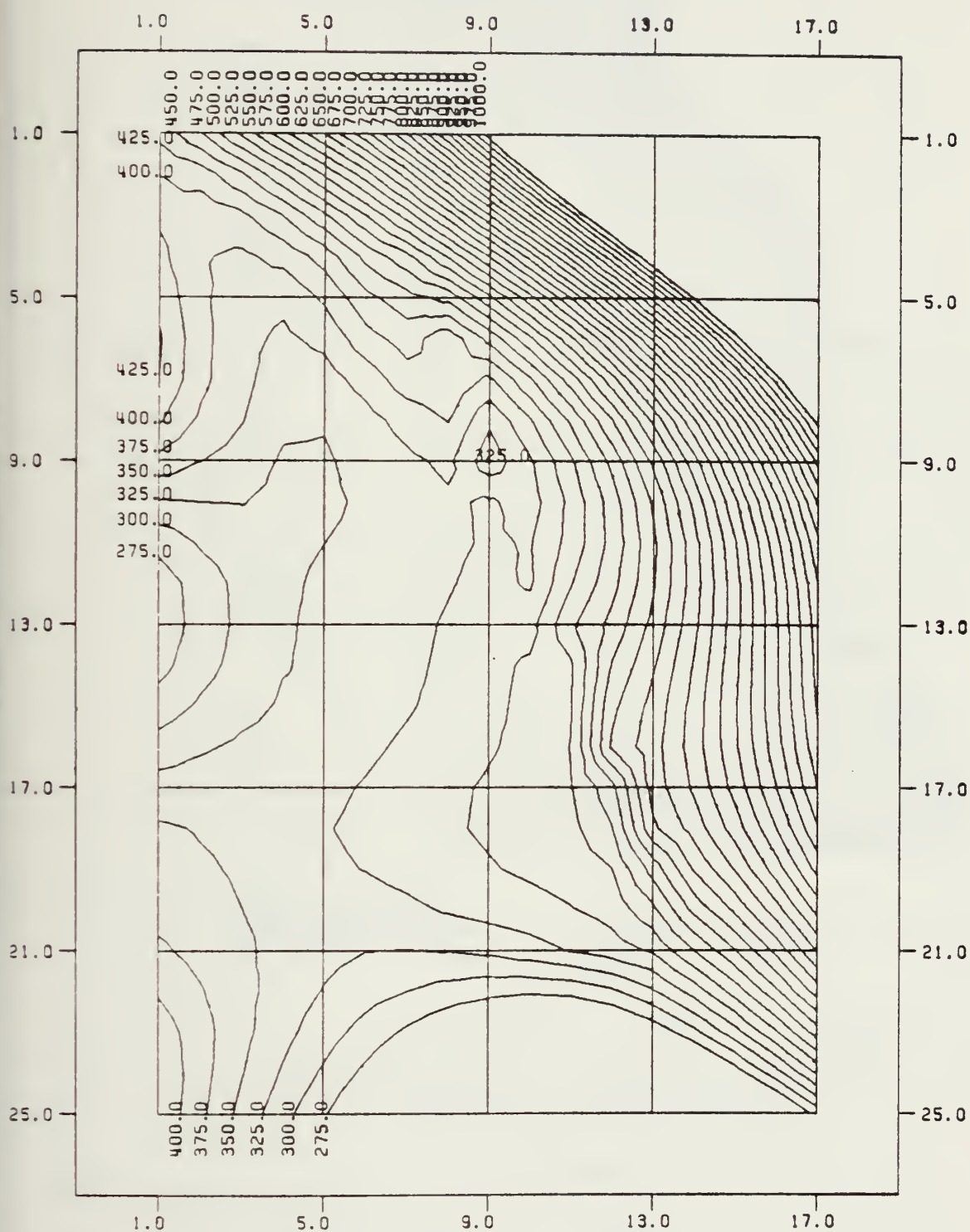
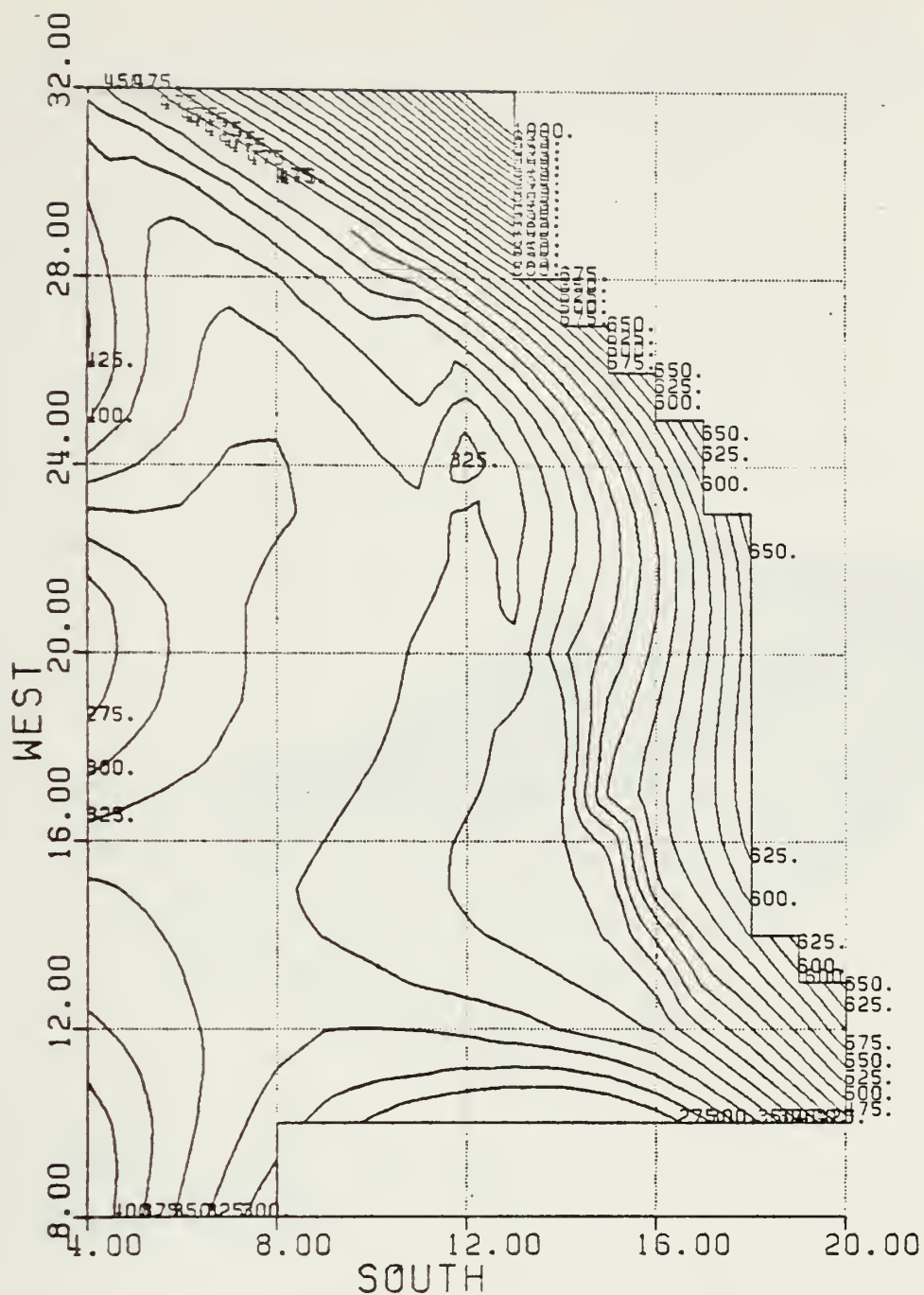


Figure 8



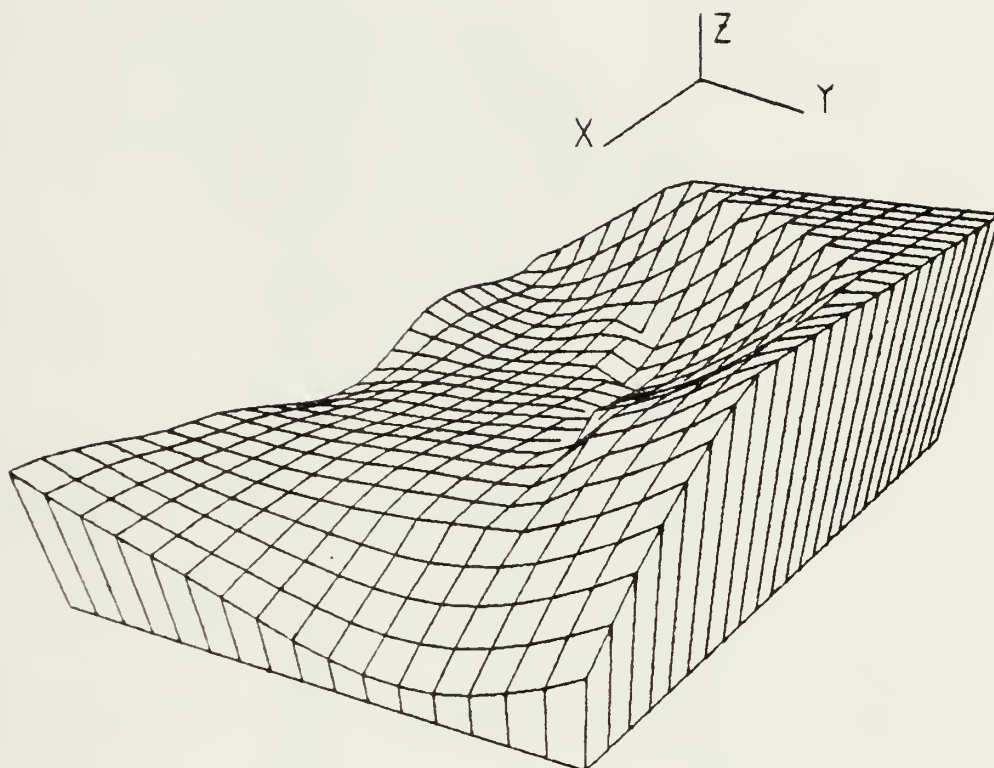


P-WAVE VEL. (NPS)  
 TRIANGLE BASED QUAD. METHOD  
 (Using subroutine CONISD)

Figure 9







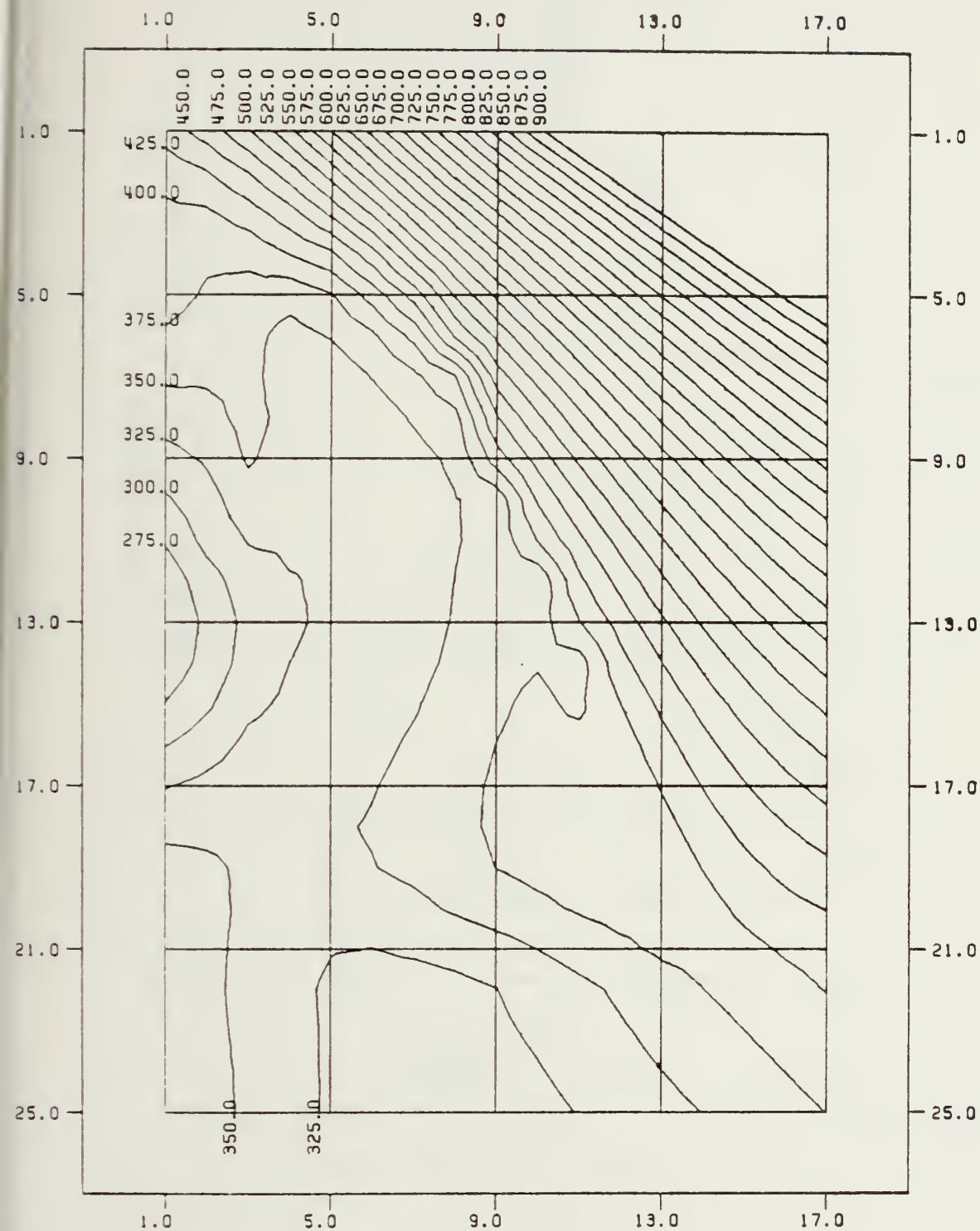
P-WAVE VEL. (NPS)  
TRIANGLE BASED QUAD. METHOD  
(Using subroutine PLT3D1)

Figure 10



# P-WAVE VEL. (NPS)

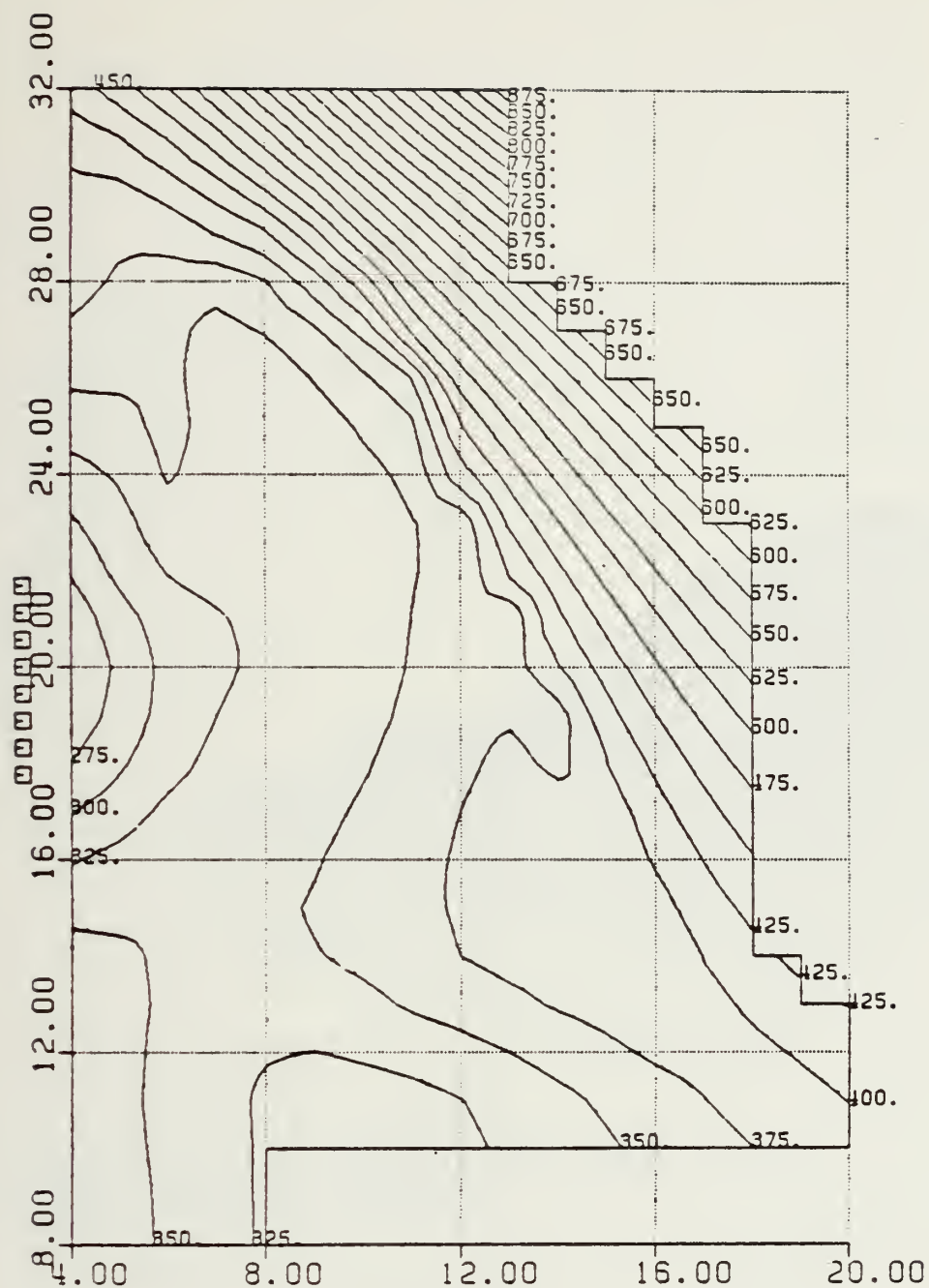
## NIELSON'S MIN. NORM NETWORK M.



(Using subroutine CONTUR)

Figure 11

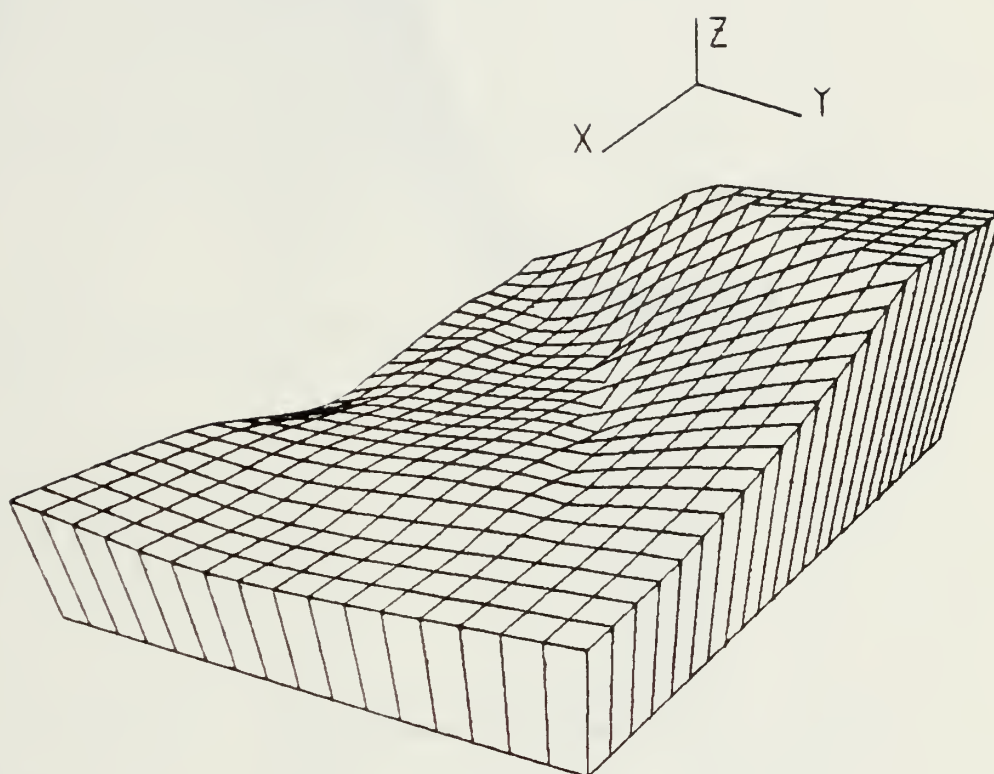




P-WAVE VEL. (NPS)  
 NIELSON'S MIN. NORM NETWORK M.  
 (Using subroutine CONISD)

Figure 12





P-WAVE VEL. (NPS)  
NIELSON'S MIN. NORM NETWORK M.  
(Using subroutine PLT3D1)

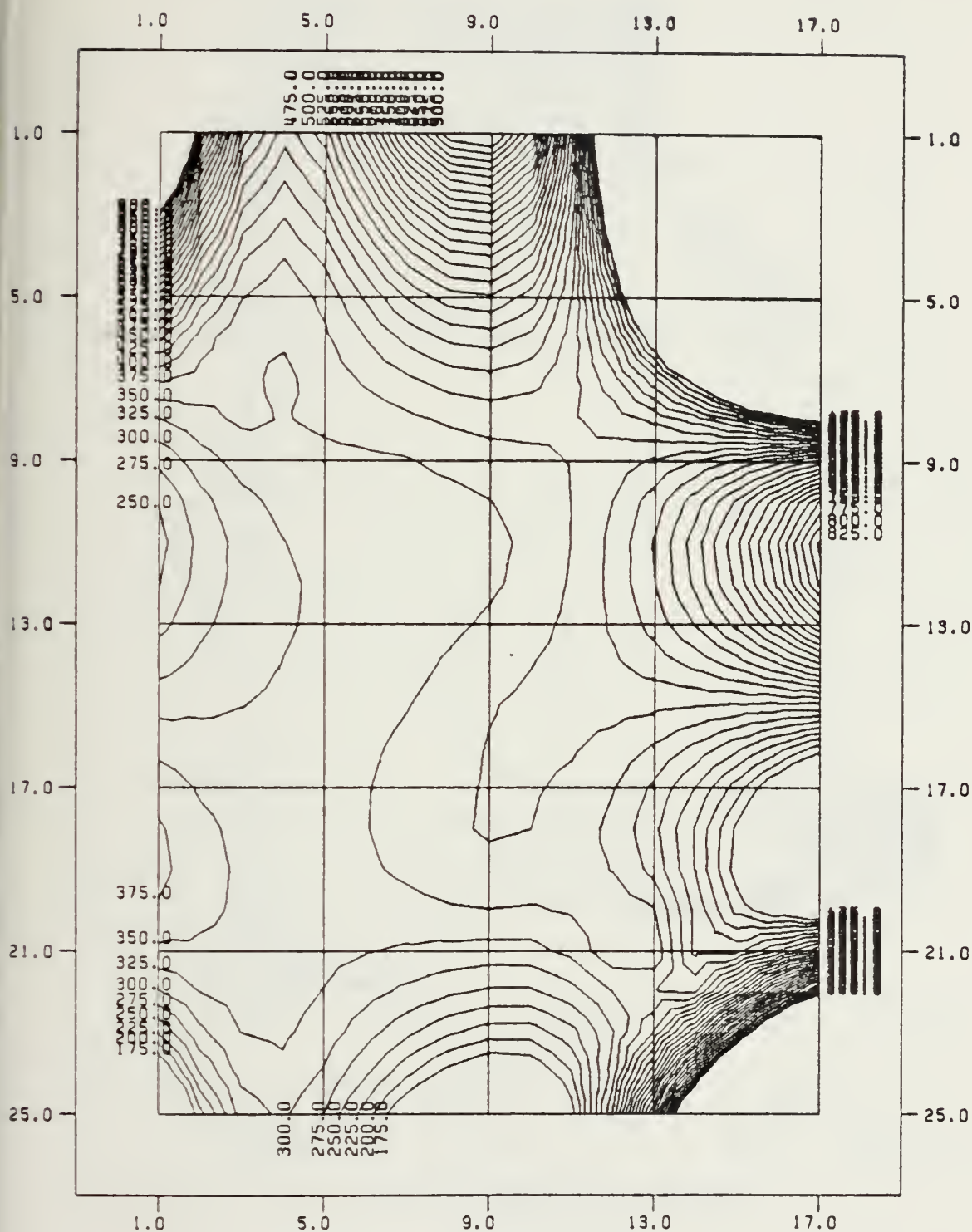
Figure 13





# P-WAVE VEL. (NPS)

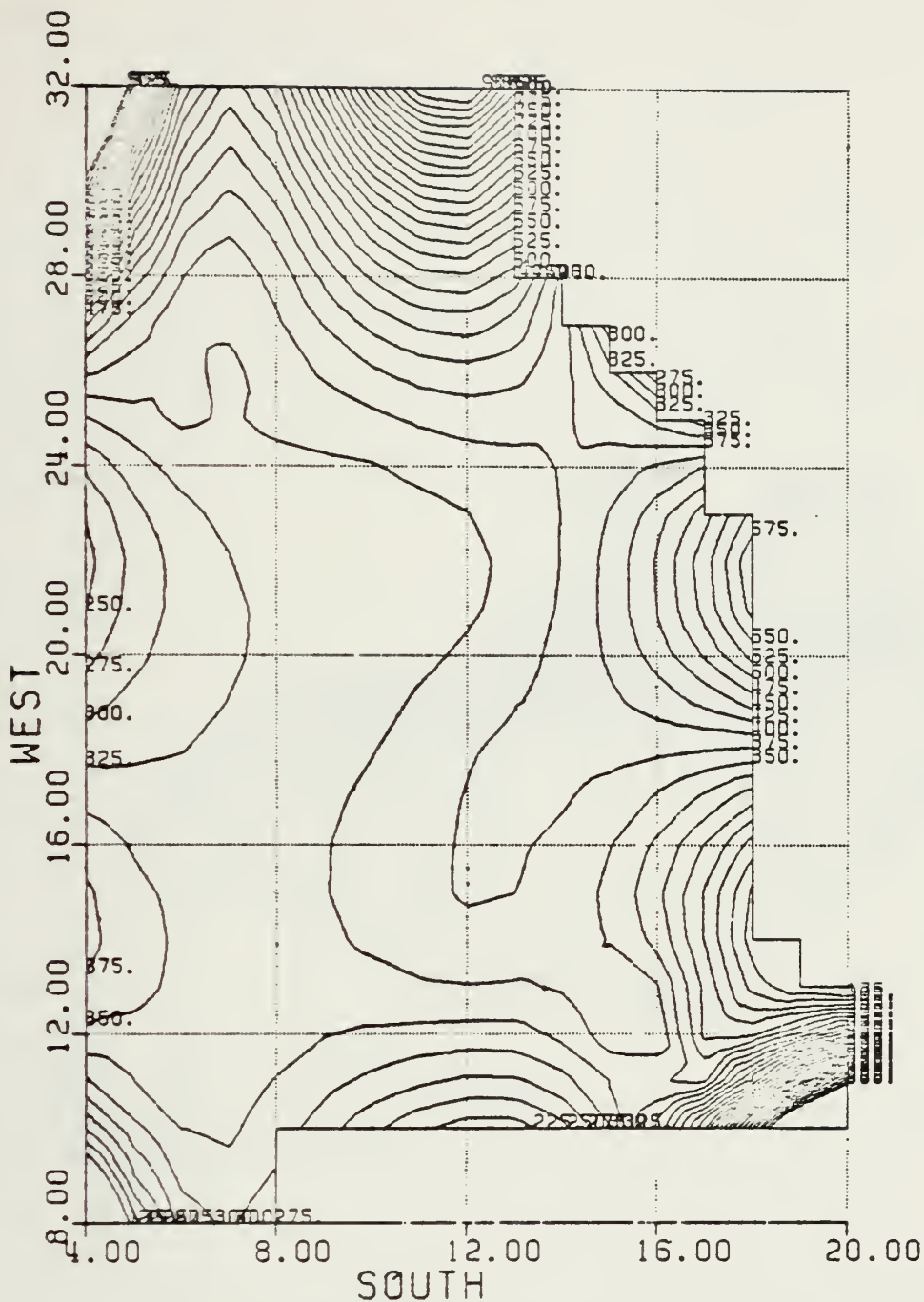
## DELTA SUM BIC. SP. METHOD



(Using subroutine CONTUR)

Figure 14



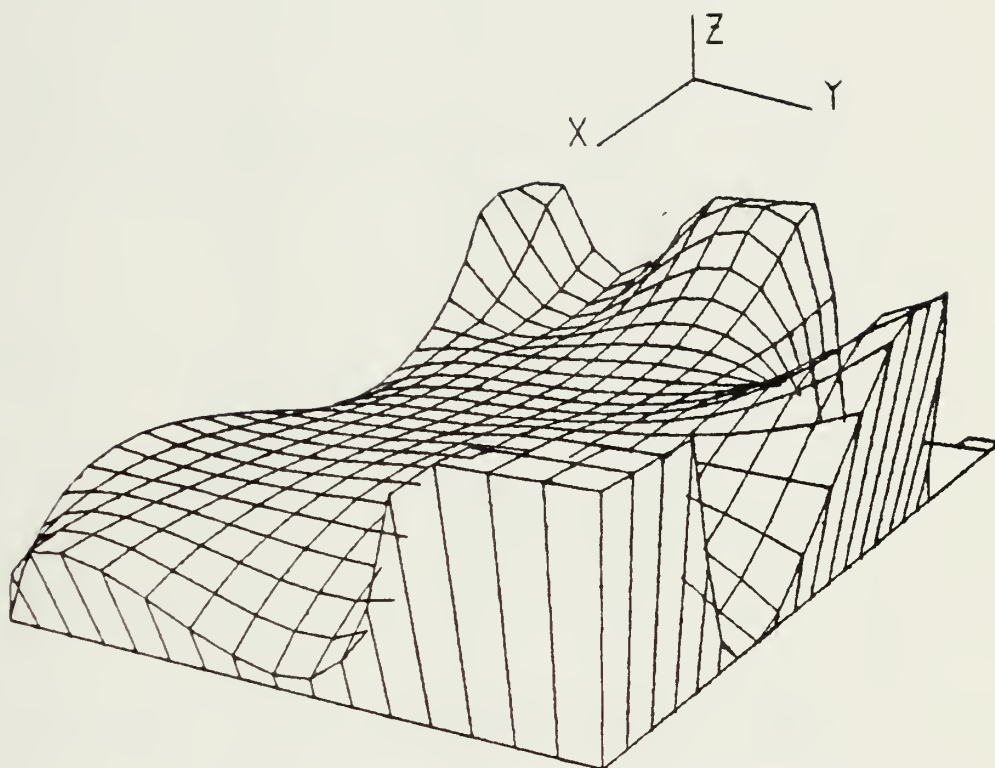


P-WAVE VEL. (NPS)  
 DELTA SUM BIC. SP. METHOD

(Using subroutine CONISD)

Figure 15





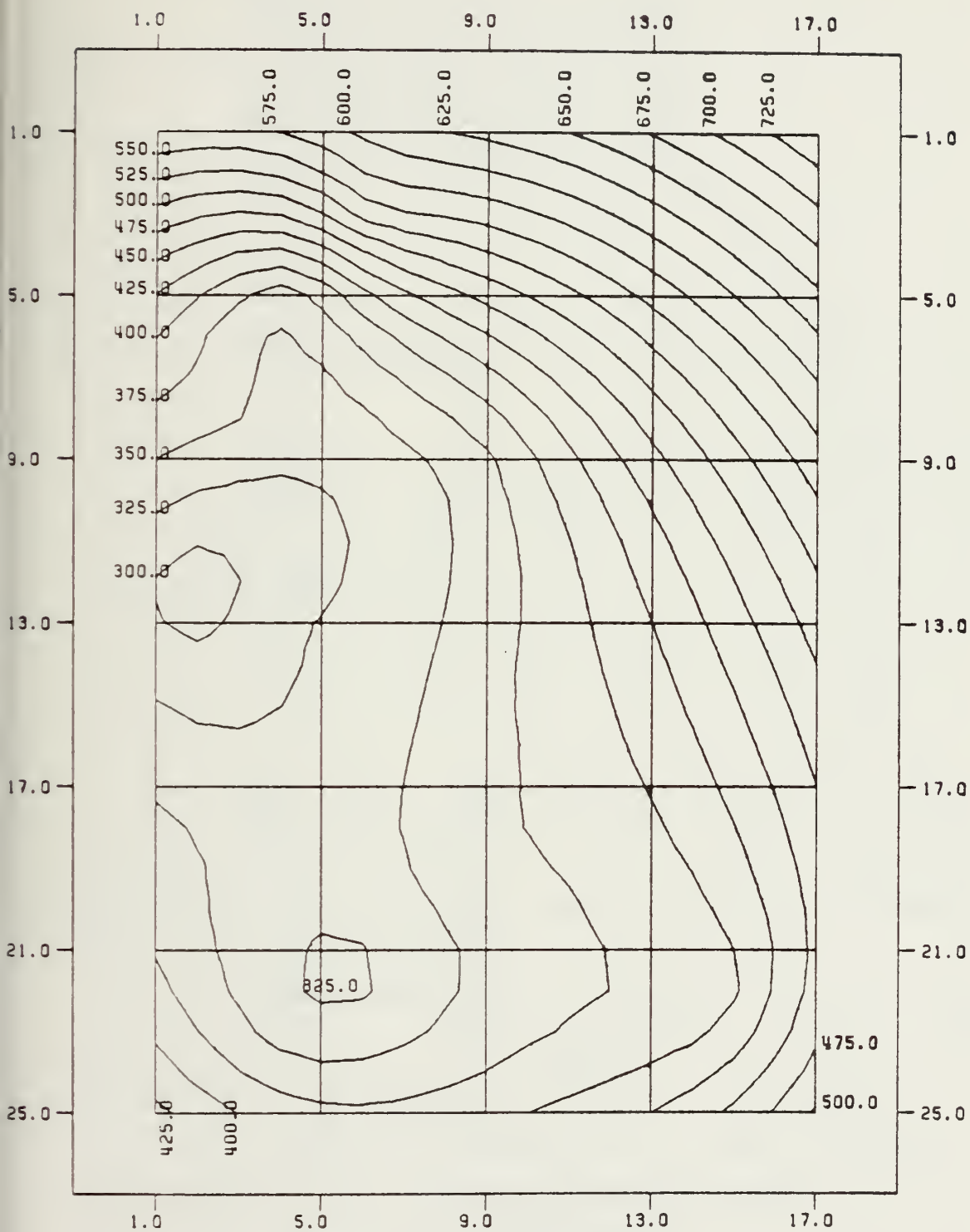
P-WAVE VEL. (NPS)  
DELTA SUM BIC. SP. METHOD  
(Using subroutine PLT3D1)

Figure 16



# P-WAVE VEL. (NPS)

## MULTIQUADRIC METHOD

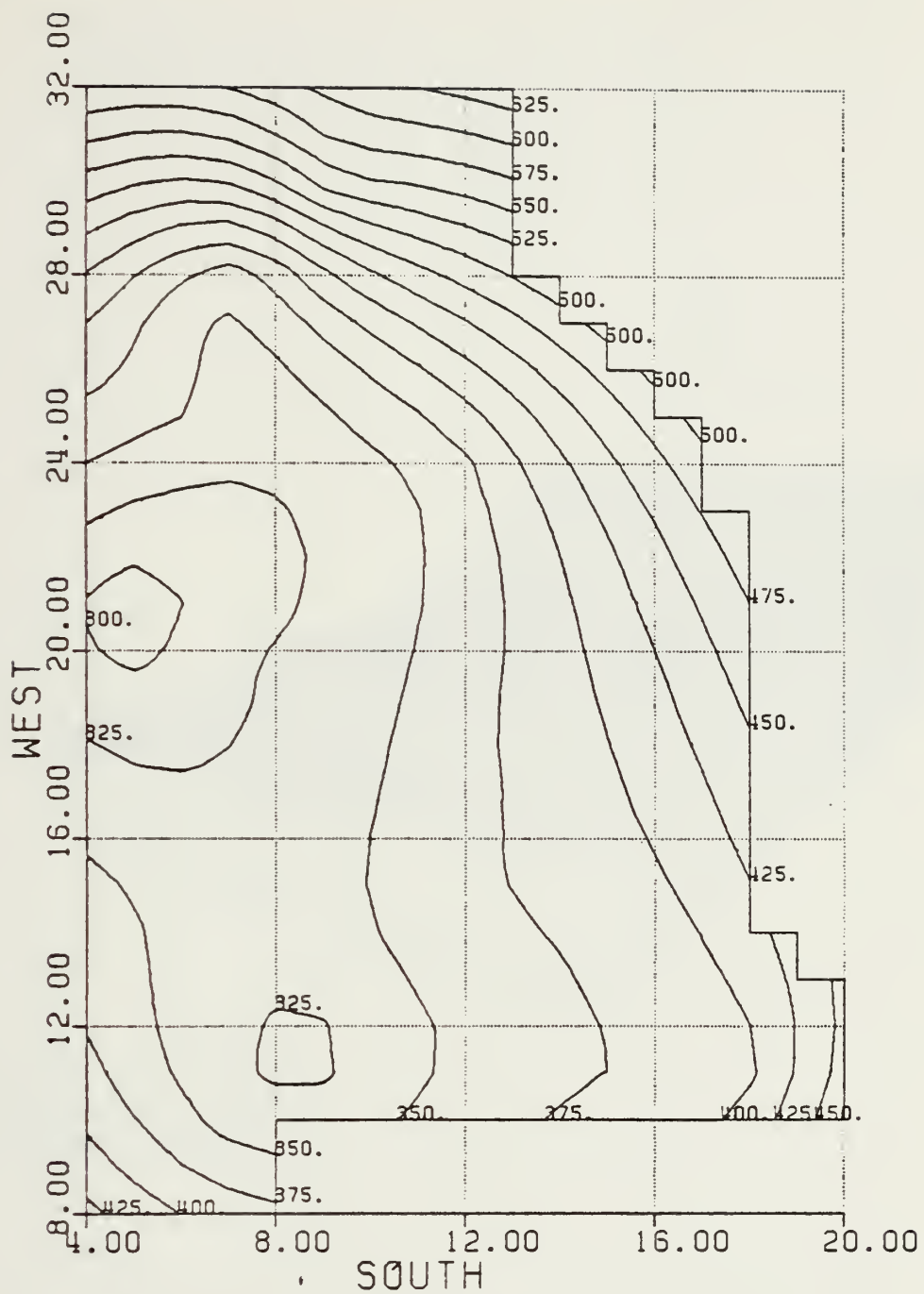


(Using subroutine CONTUR)

Figure 17





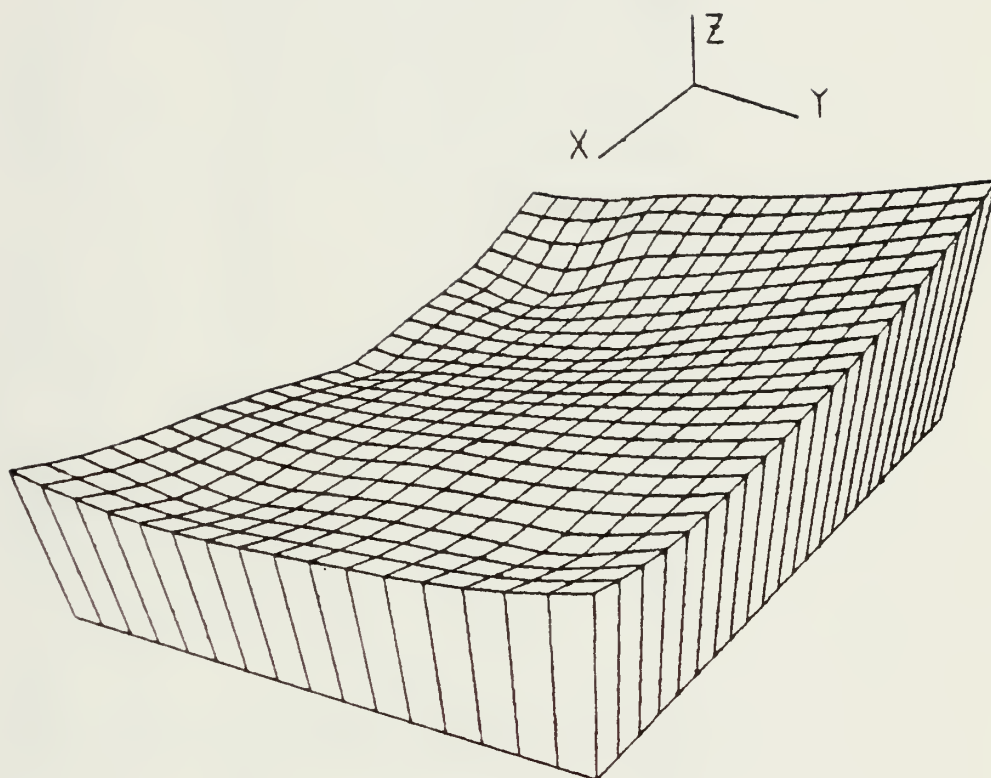


P-WAVE VEL. (NPS)  
MULTIQUADRIC METHOD

(Using subroutine CONISD)

Figure 18



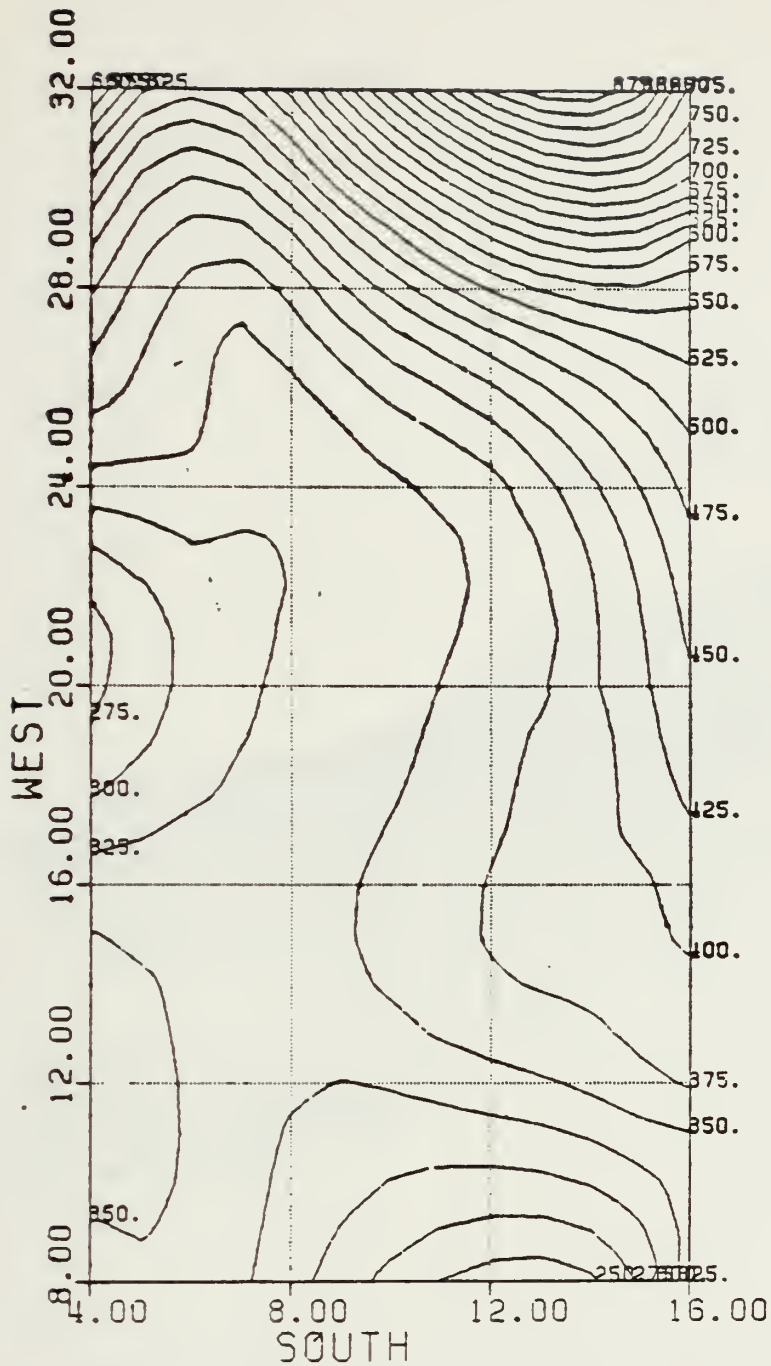


P-WAVE VEL. (NPS)  
MULTIQUADRIC METHOD  
(Using subroutine PLT3D1)

Figure 19



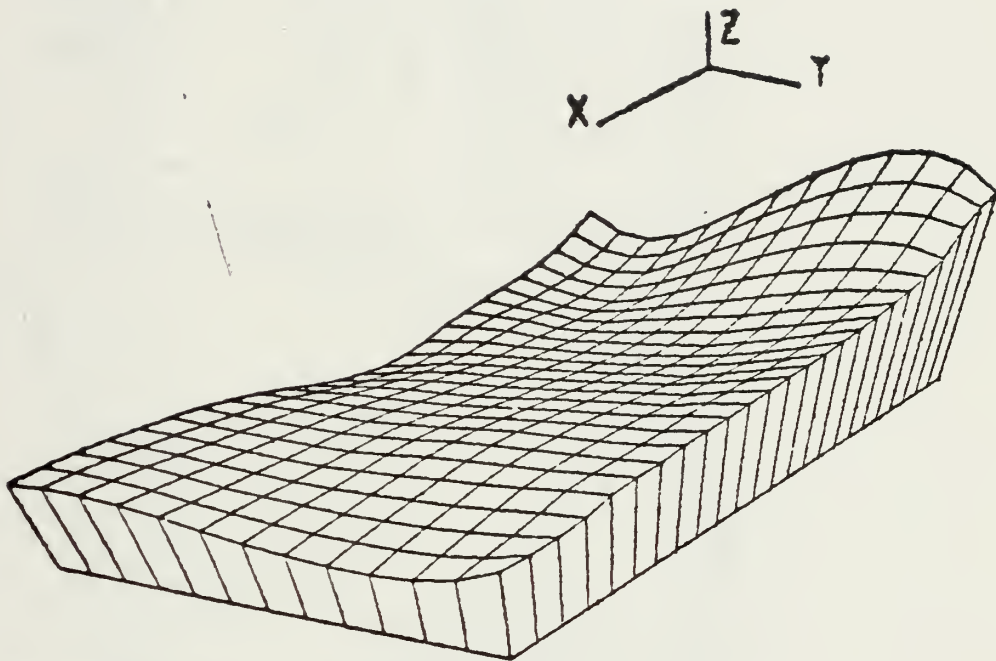
APPENDIX D



SEISMIC P-WAVE VELOCITIES  
RALSTON VALLEY, NEVADA  
(Using subroutine CONISD)

Figure 20



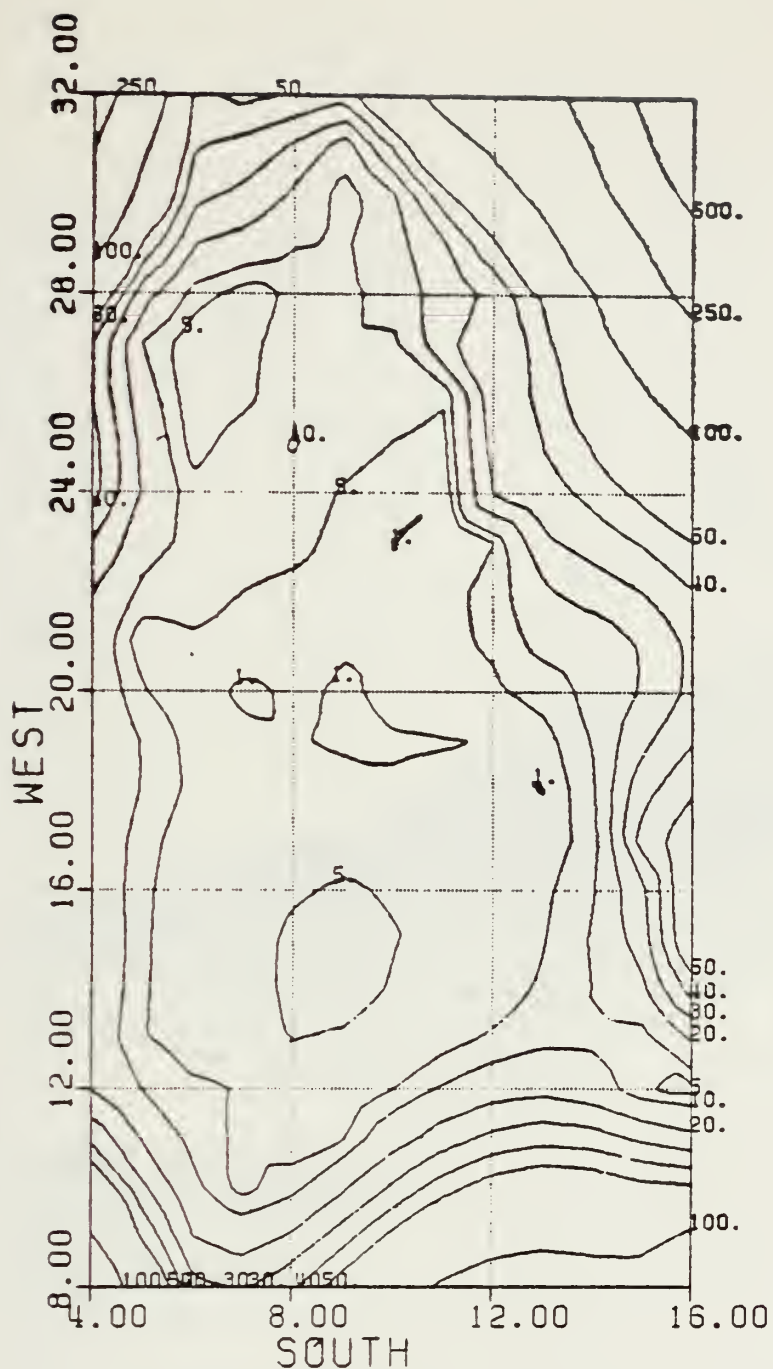


SEISMIC P-WAVE VELOCITIES  
RALSTON VALLEY, NEVADA  
(Using subroutine PLT3D1)

Figure 21



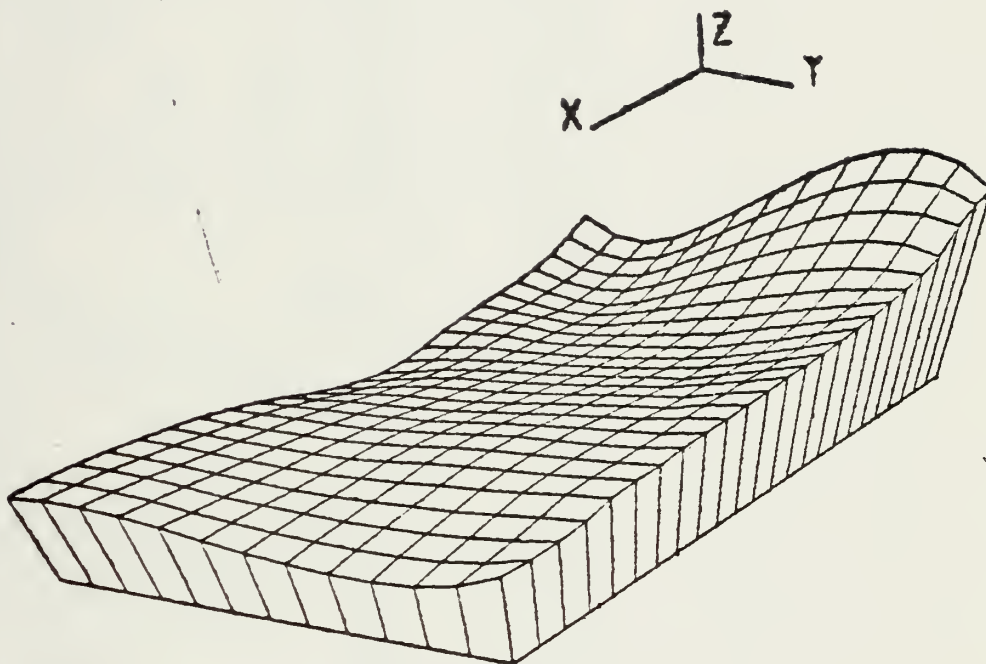




STANDARD DEVIATIONS FOR  
SEISMIC P-WAVE VELOCITIES  
(Using subroutine CONISD)

Figure 22





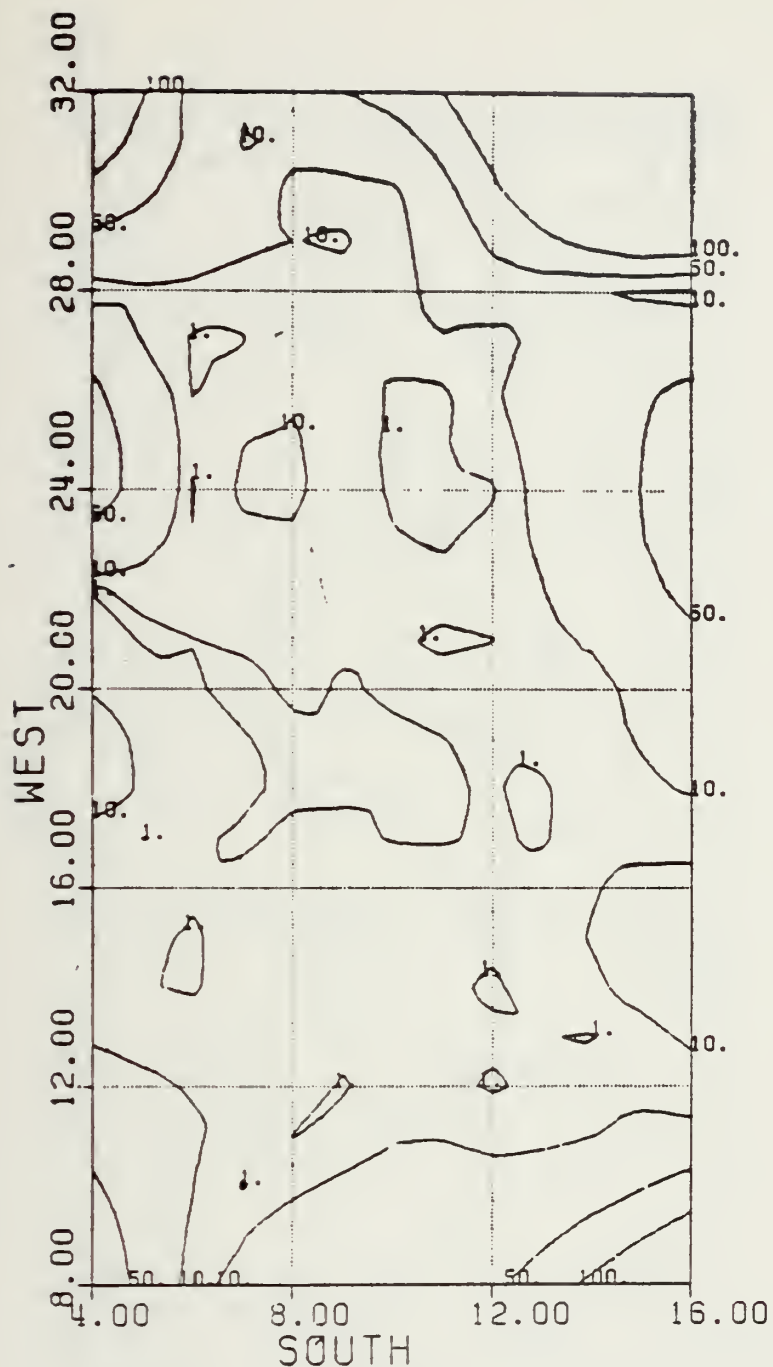
SEISMIC P-WAVE VELOCITIES  
RALSTON VALLEY, NEVADA

(Using subroutine PLT3D1)

Figure 23.



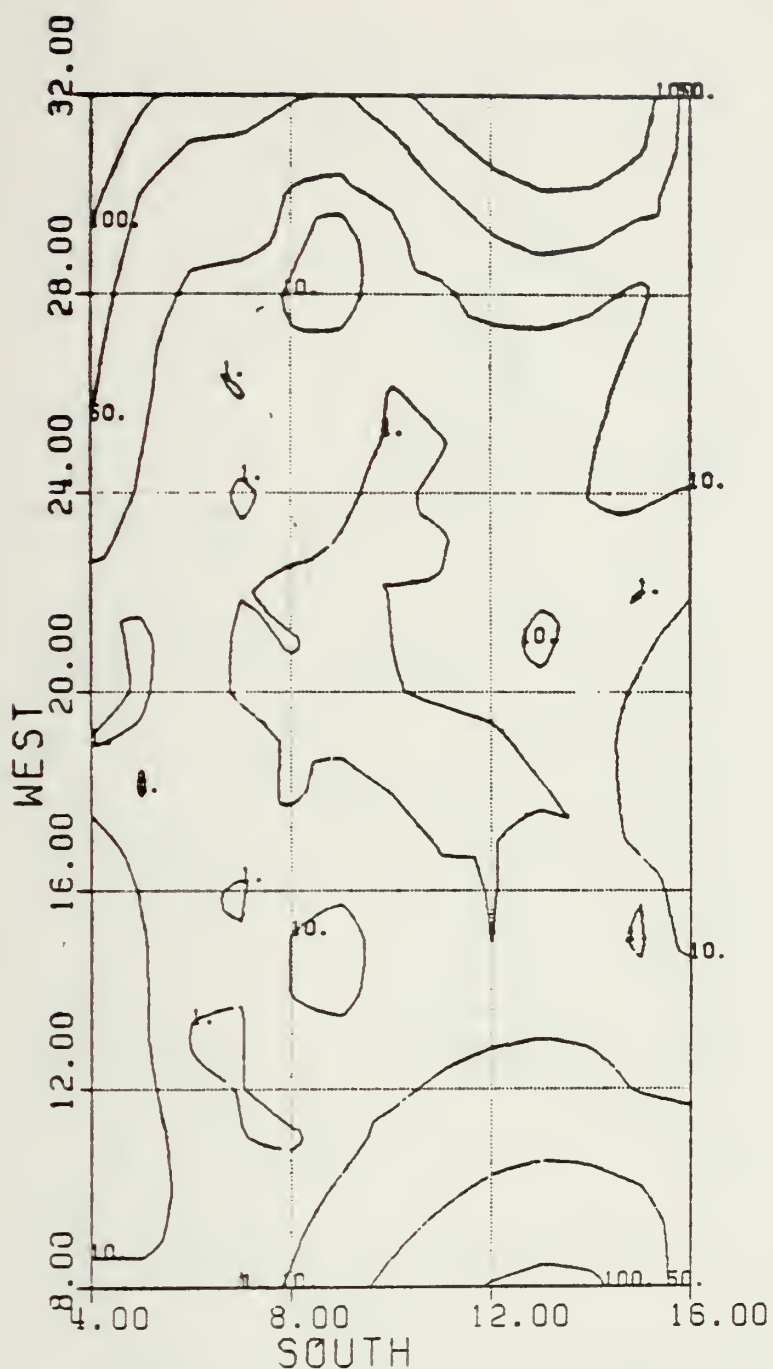
# APPENDIX E



ABS ( QUAD. SHEP. M. - MEAN )  
 SEISMIC P-WAVE VELOCITIES  
 (Using subroutine CONISD)

Figure 24





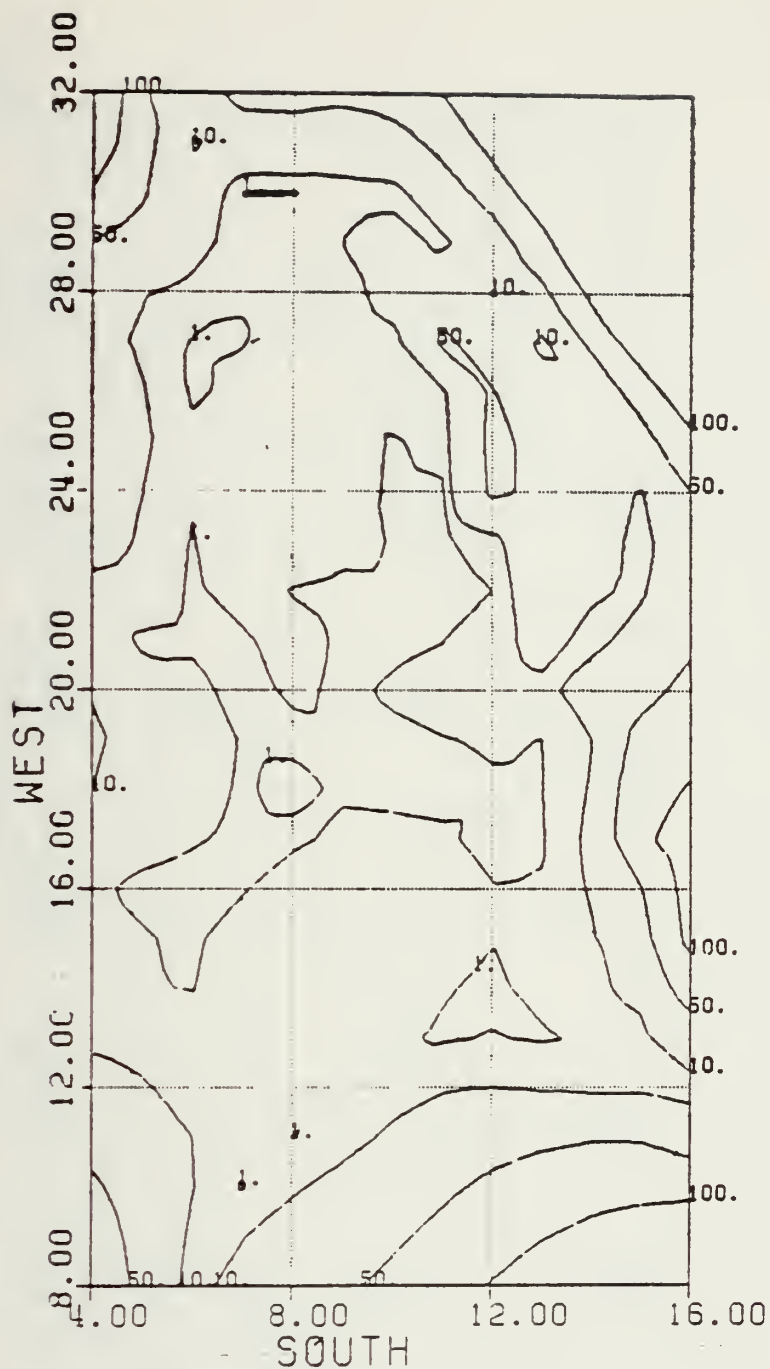
ABS. ( LOCAL THIN M. - MEAN )  
SEISMIC P-WAVE VELOCITIES

(Using subroutine CONISD)

Figure 25



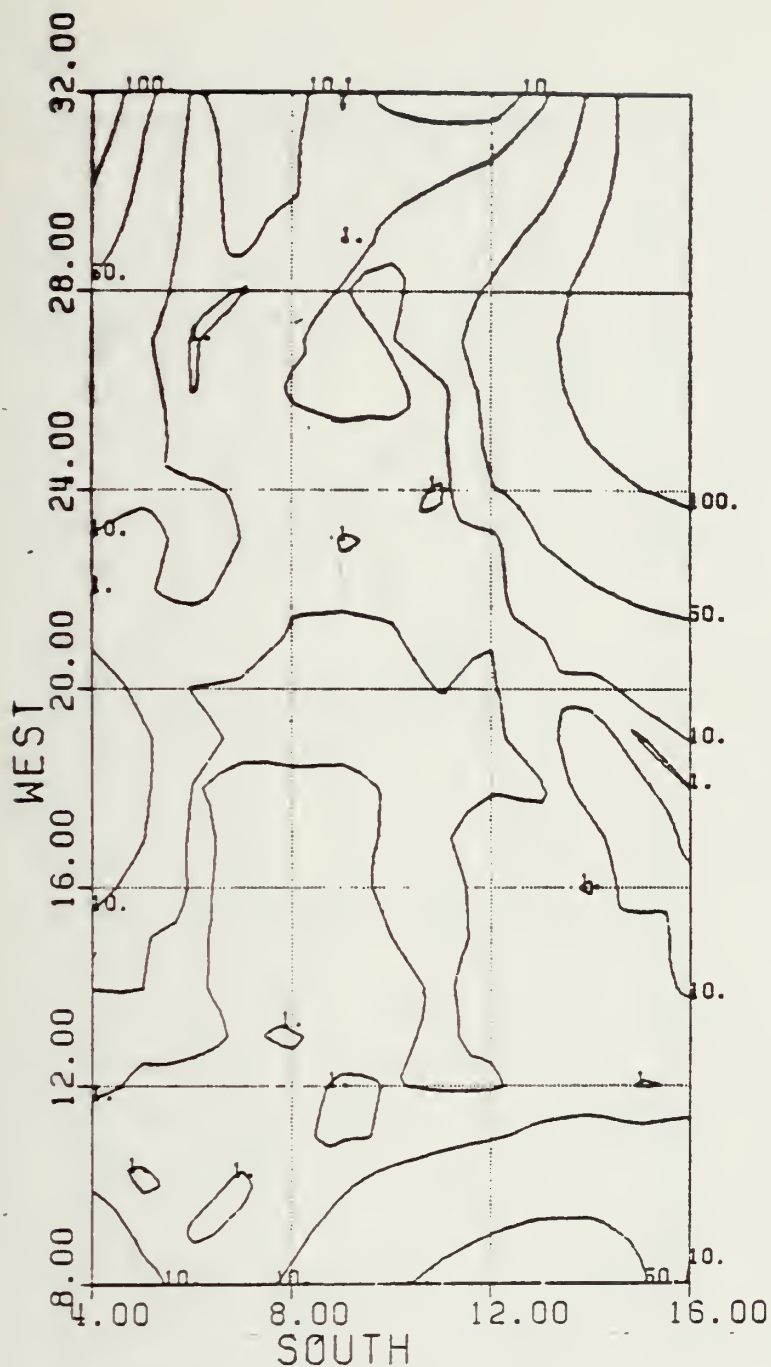




ABS ( TRI.BASED M. - MEAN )  
 SEISMIC P-WAVE VELOCITIES  
 (Using subroutine CONISD)

Figure 26

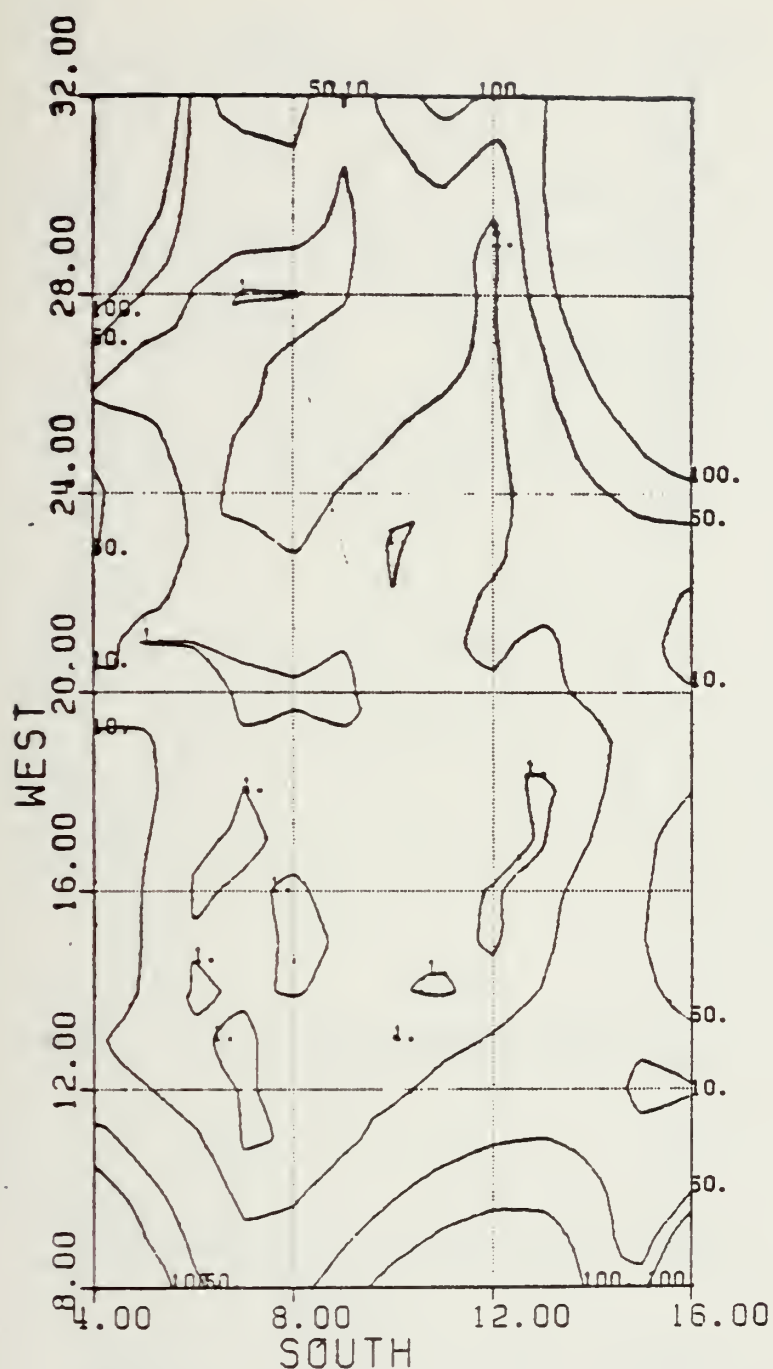




ABS ( NIELSON MIN. M. - MEAN )  
 SEISMIC P-WAVE VELOCITIES  
 (Using subroutine CONISD)

Figure 27.



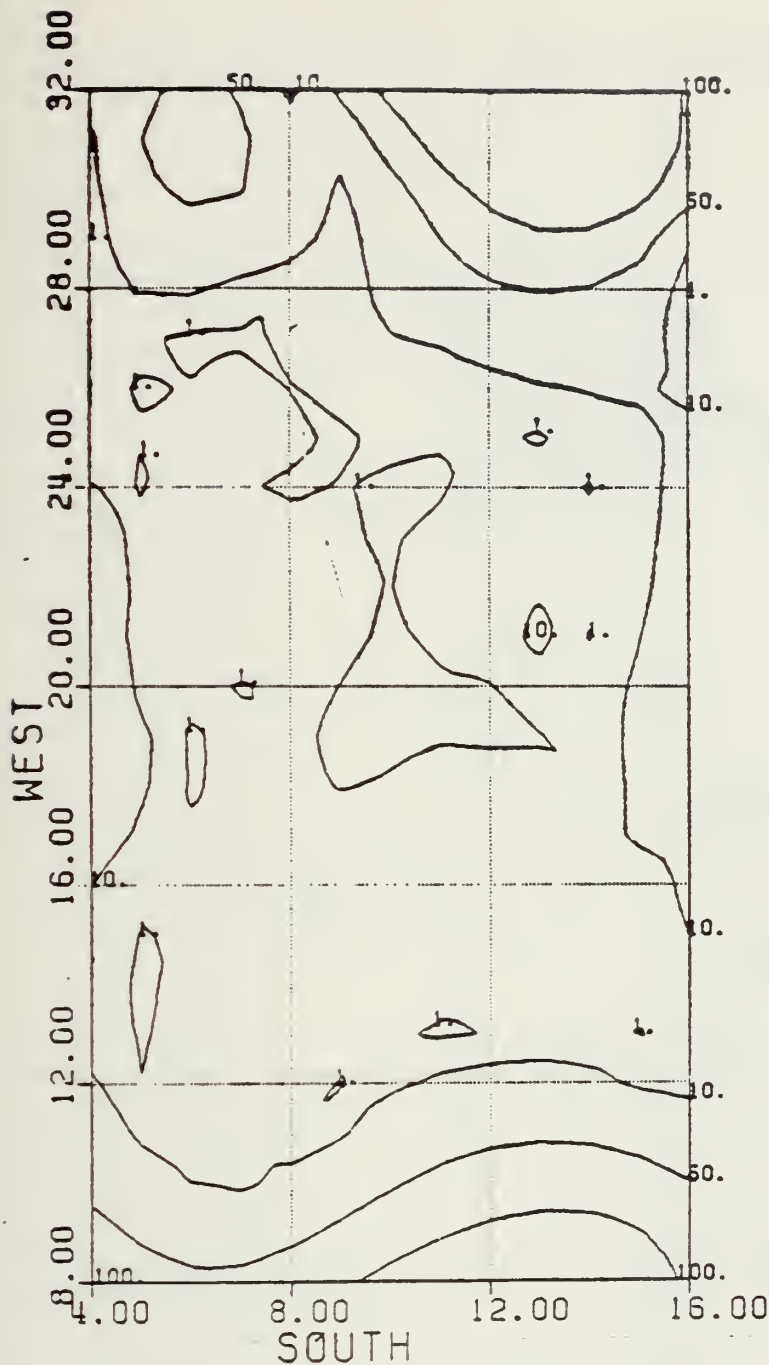


ABS. ( DELTA SUM M. - MEAN )  
SEISMIC P-WAVE VELOCITIES

(Using subroutine CONISD)

Figure 28





ABS ( MULTIQUADRIC M. - MEAN )  
 SEISMIC P-WAVE VELOCITIES  
 (Using subroutine CONISD)

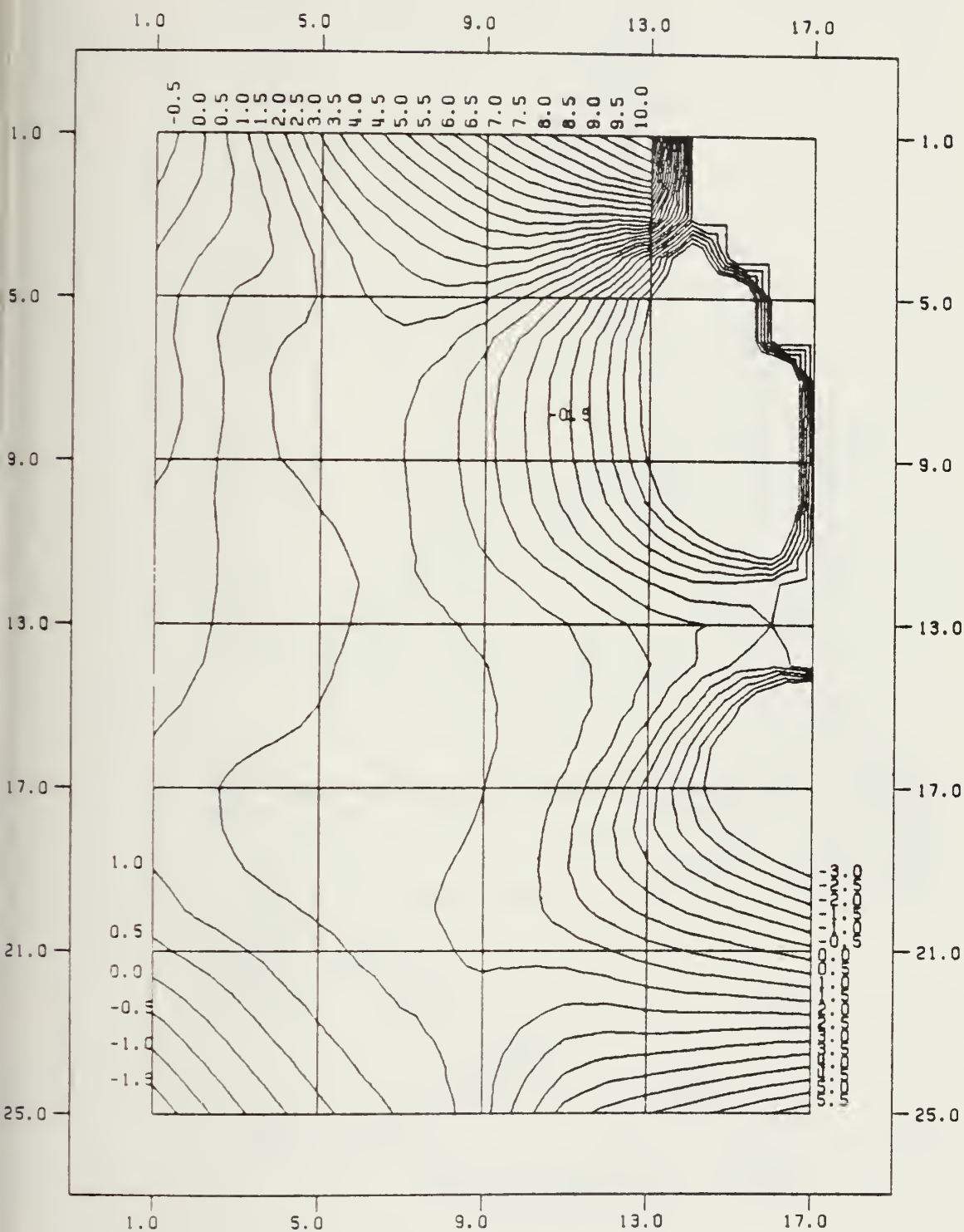
Figure 29





# DEPTH (M)

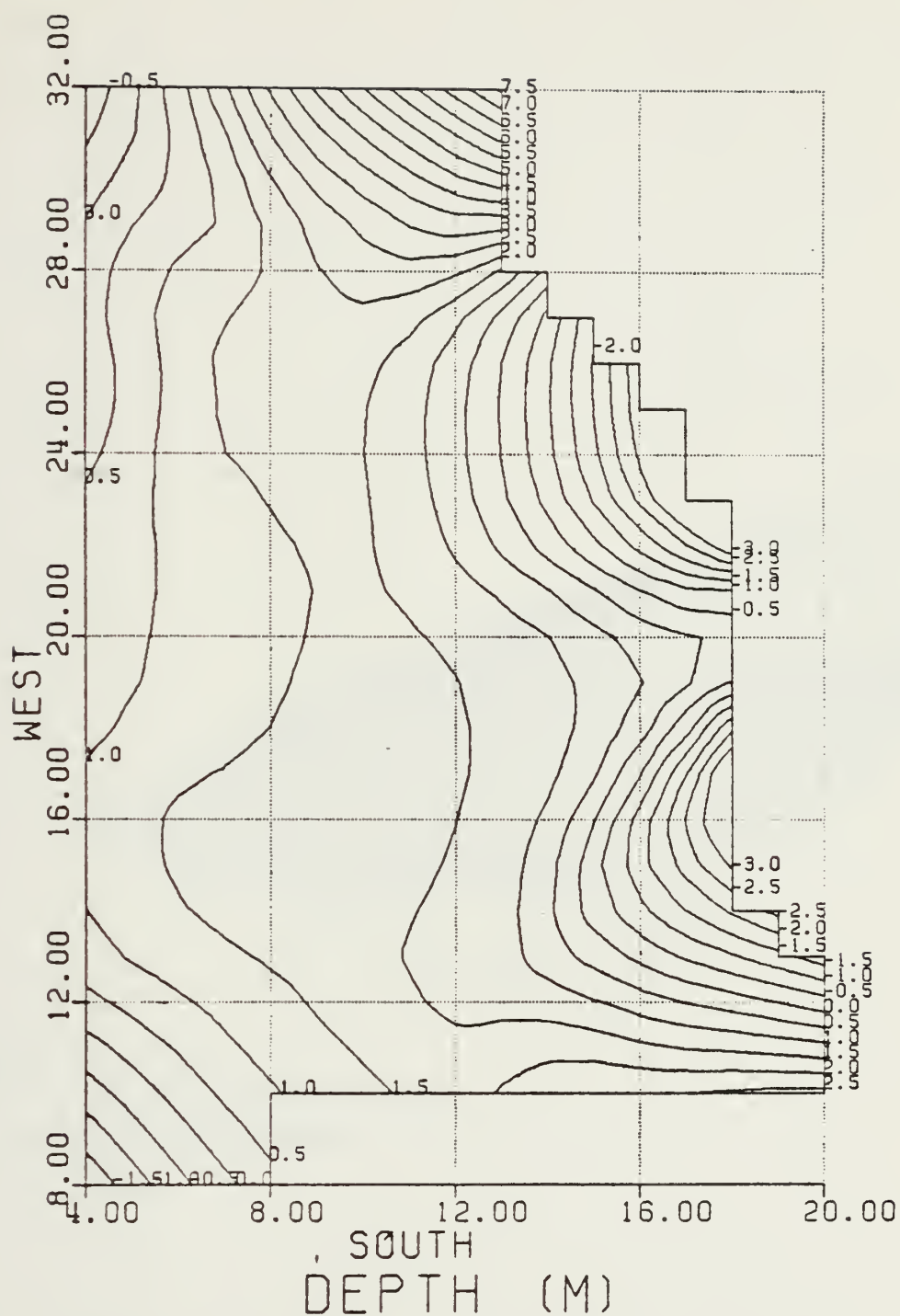
## QUADRATIC SHEPARDS METHOD



(Using subroutine CONTUR)

Figure 30



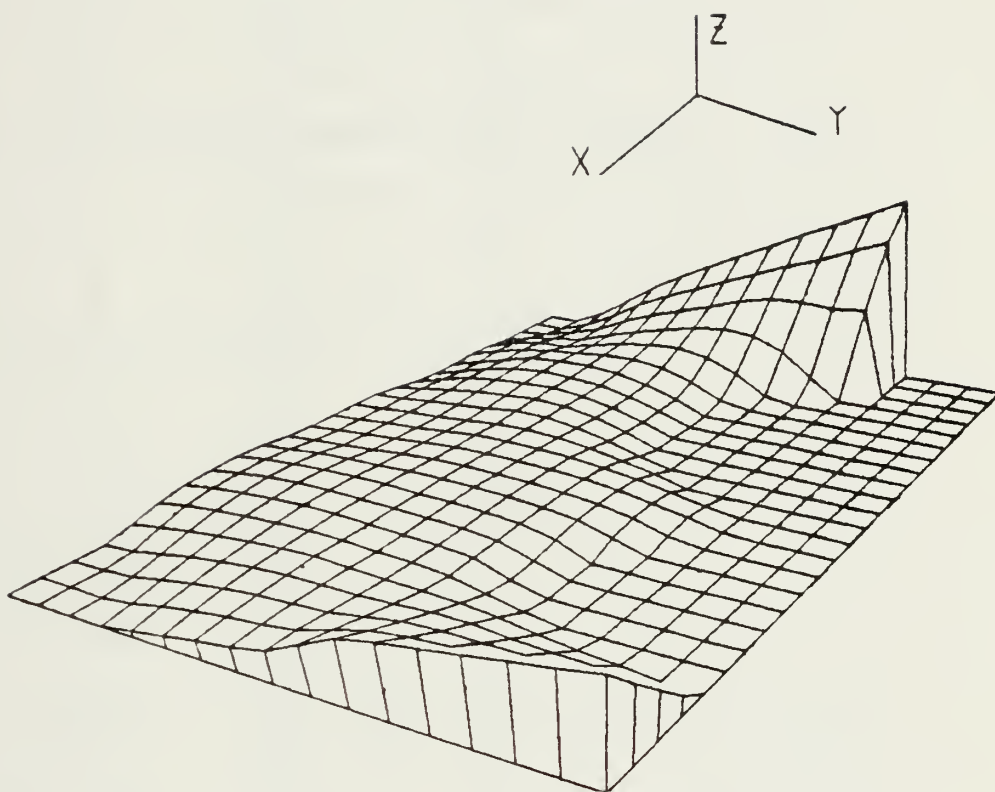


QUADRATIC SHEPARDS METHOD

(Using subroutine CONISD)

Figure 31



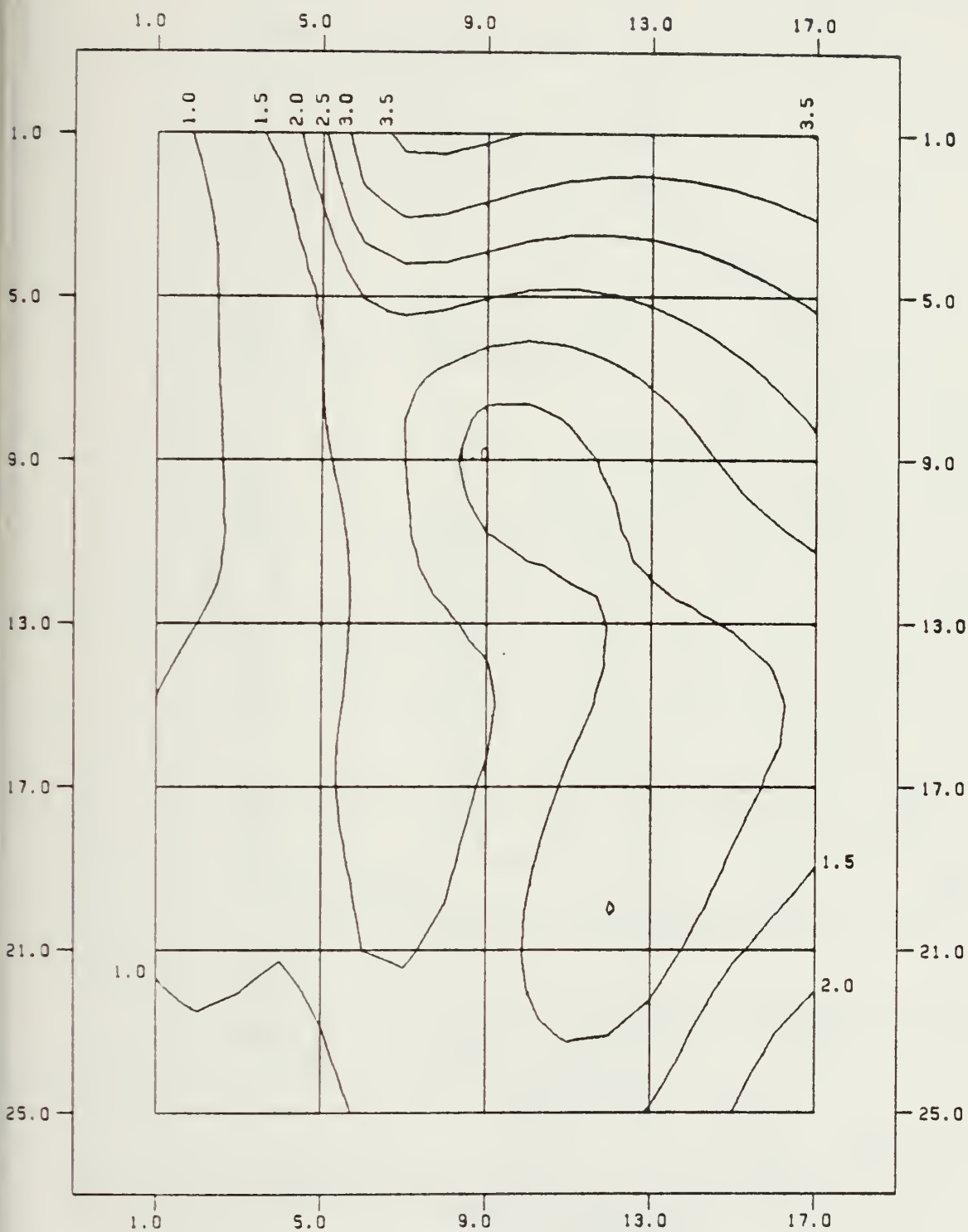


DEPTH (M)  
QUADRATIC SHEPARDS METHOD  
(Using subroutine PLT3D1)

Figure 32



DEPTH (M)  
LOCAL THIN PLATE SPLINE M.

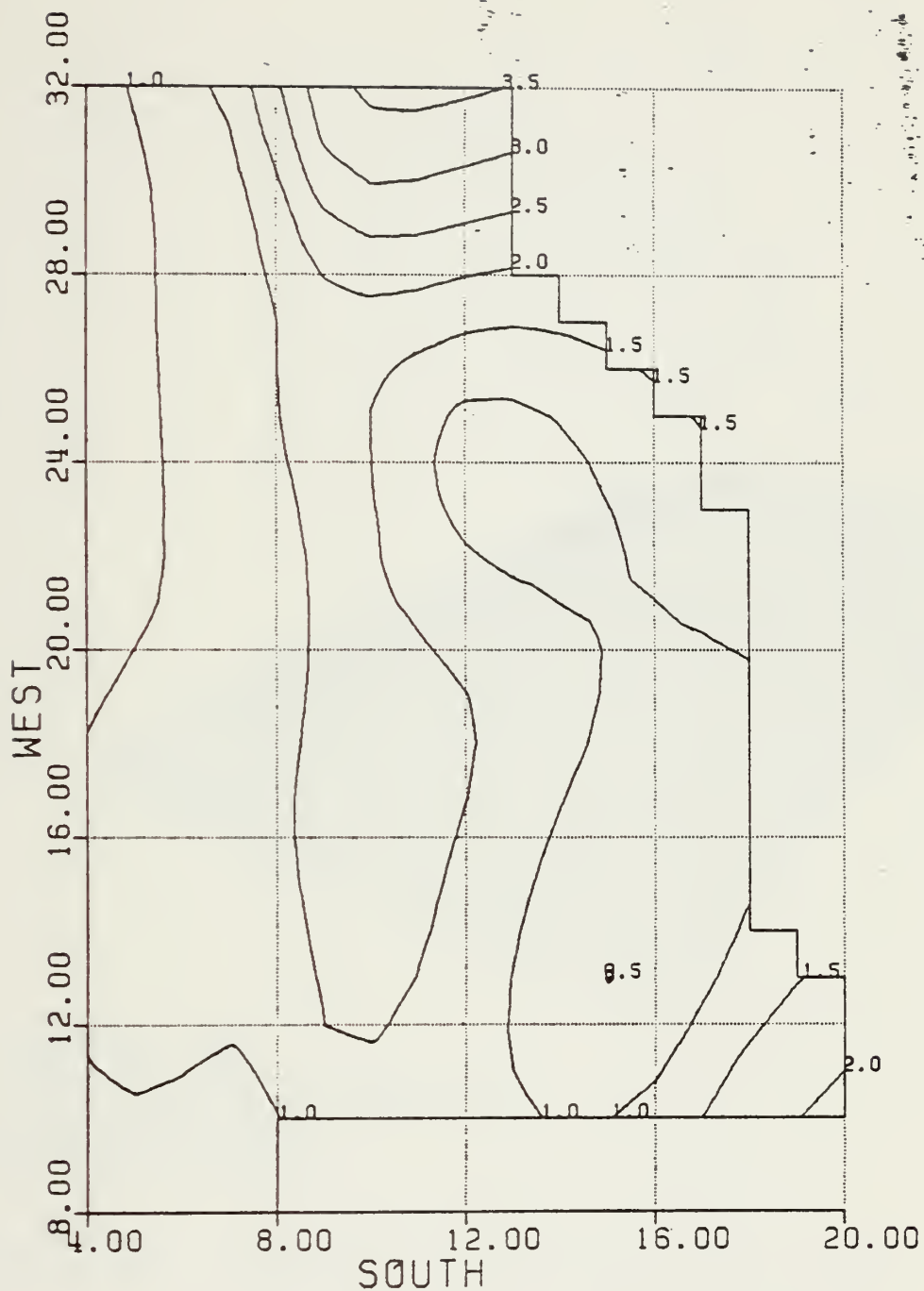


(Using subroutine CONTUR)

Figure 33





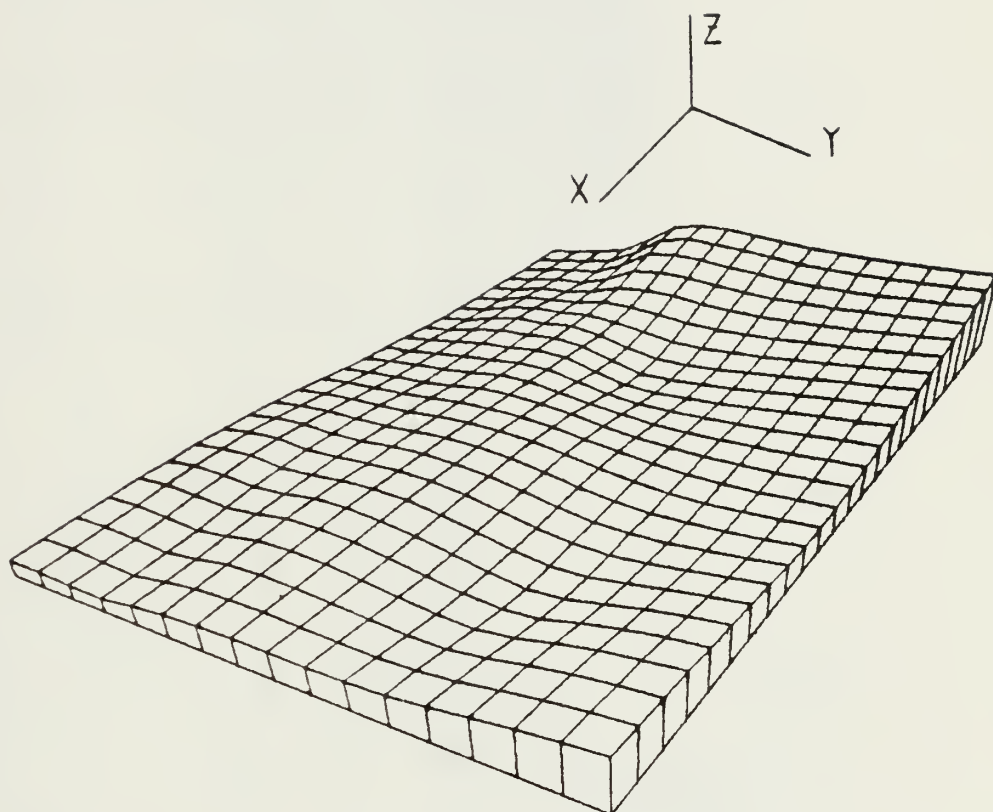


SOUTH  
 DEPTH (M)  
 LOCAL THIN PLATE SPLINE M.

(Using subroutine CONISD)

Figure 34





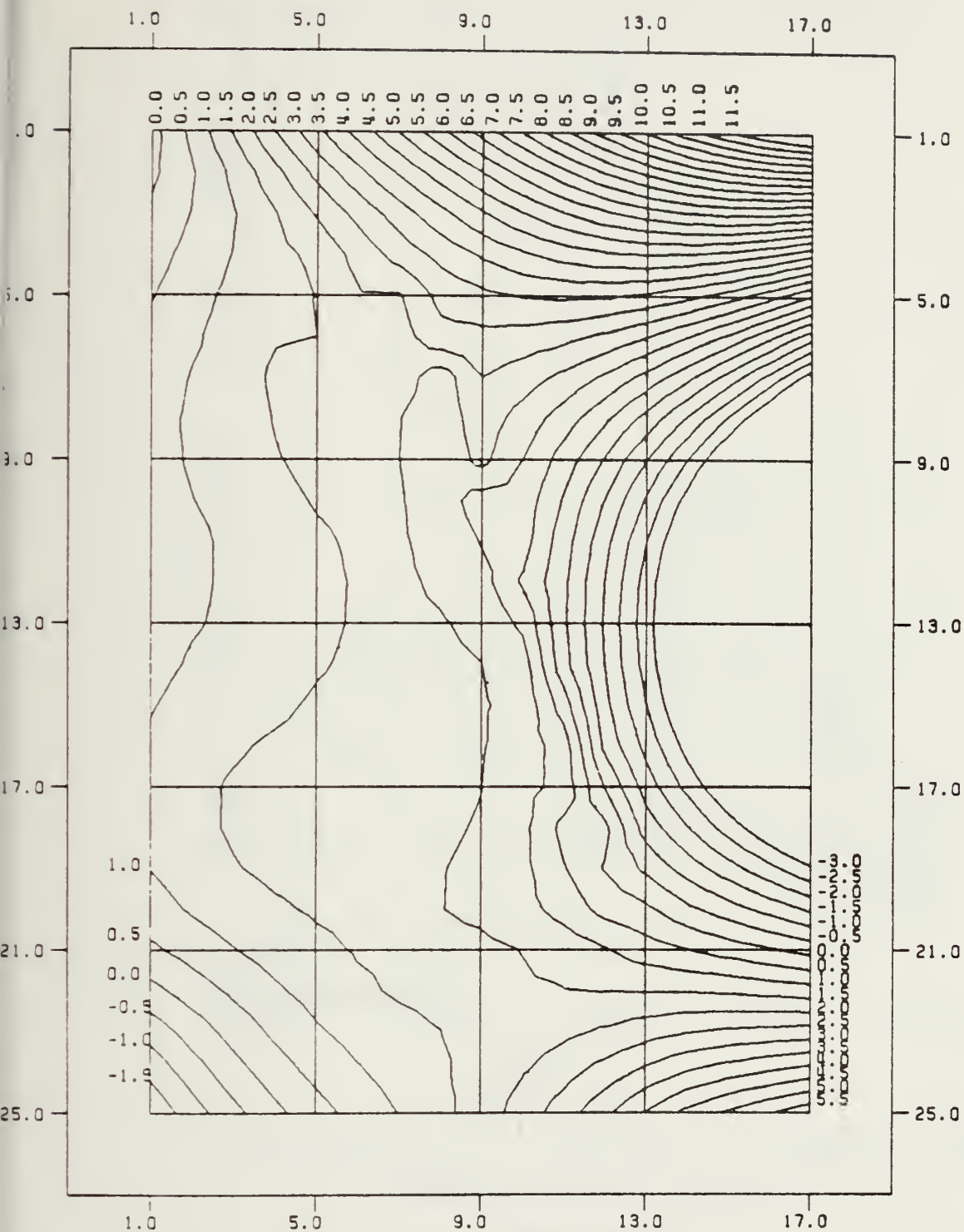
DEPTH (M)  
LOCAL THIN PLATE SPLINE M.  
(Using subroutine PLT3D1)

Figure 35



DEPTH (M)

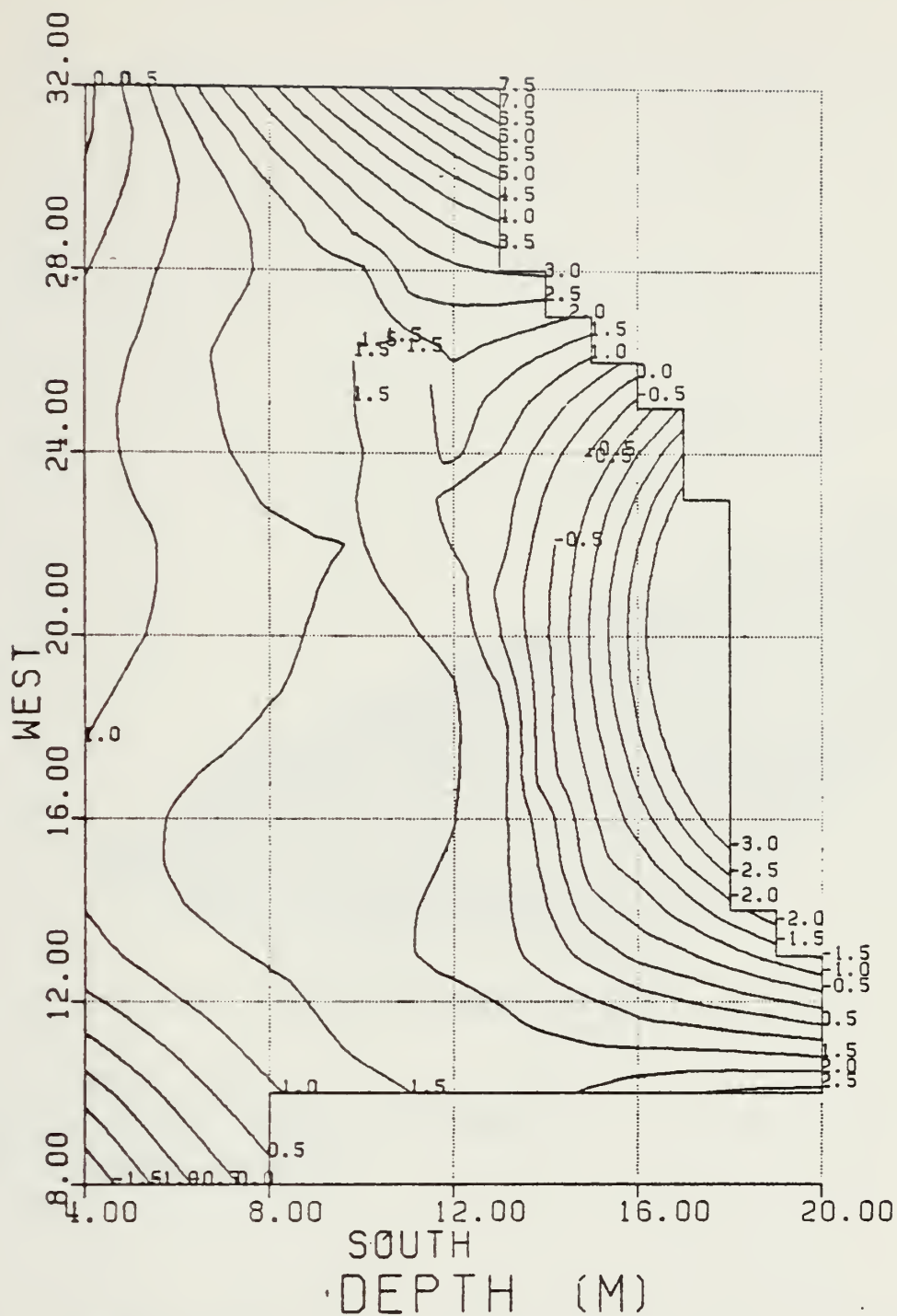
TRIANGLE BASED QUAD. METHOD



(Using subroutine CONTUR)

Figure 36



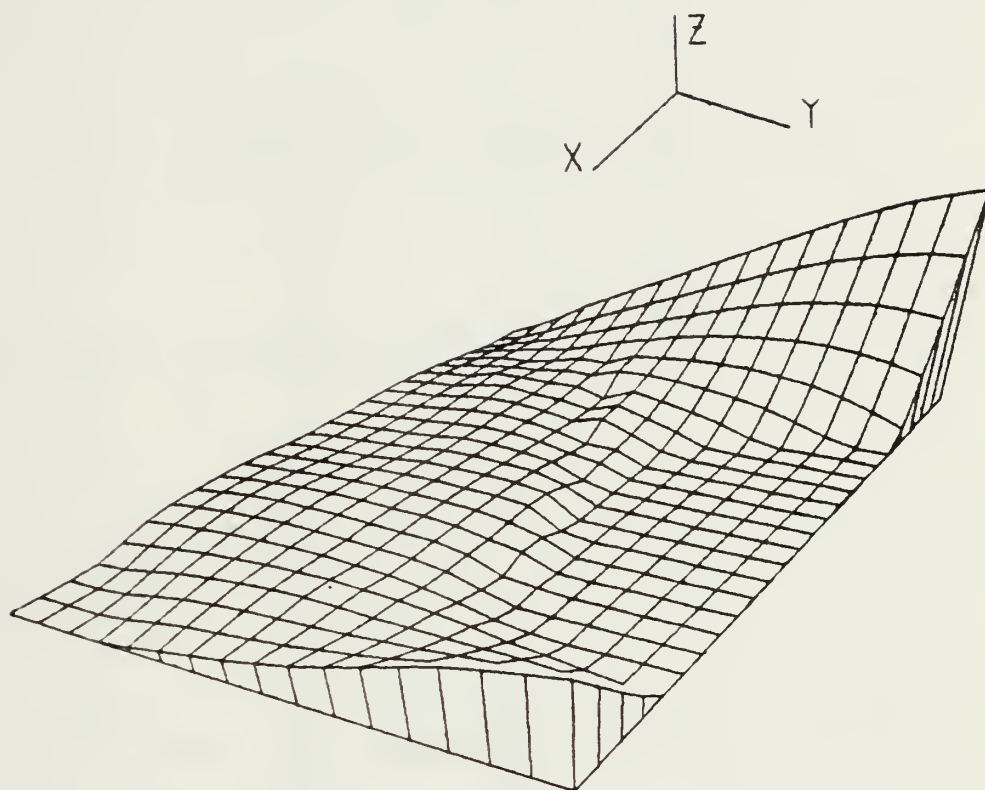


TRIANGLE BASED QUAD. METHOD  
(Using subroutine CONSID)

Figure 37







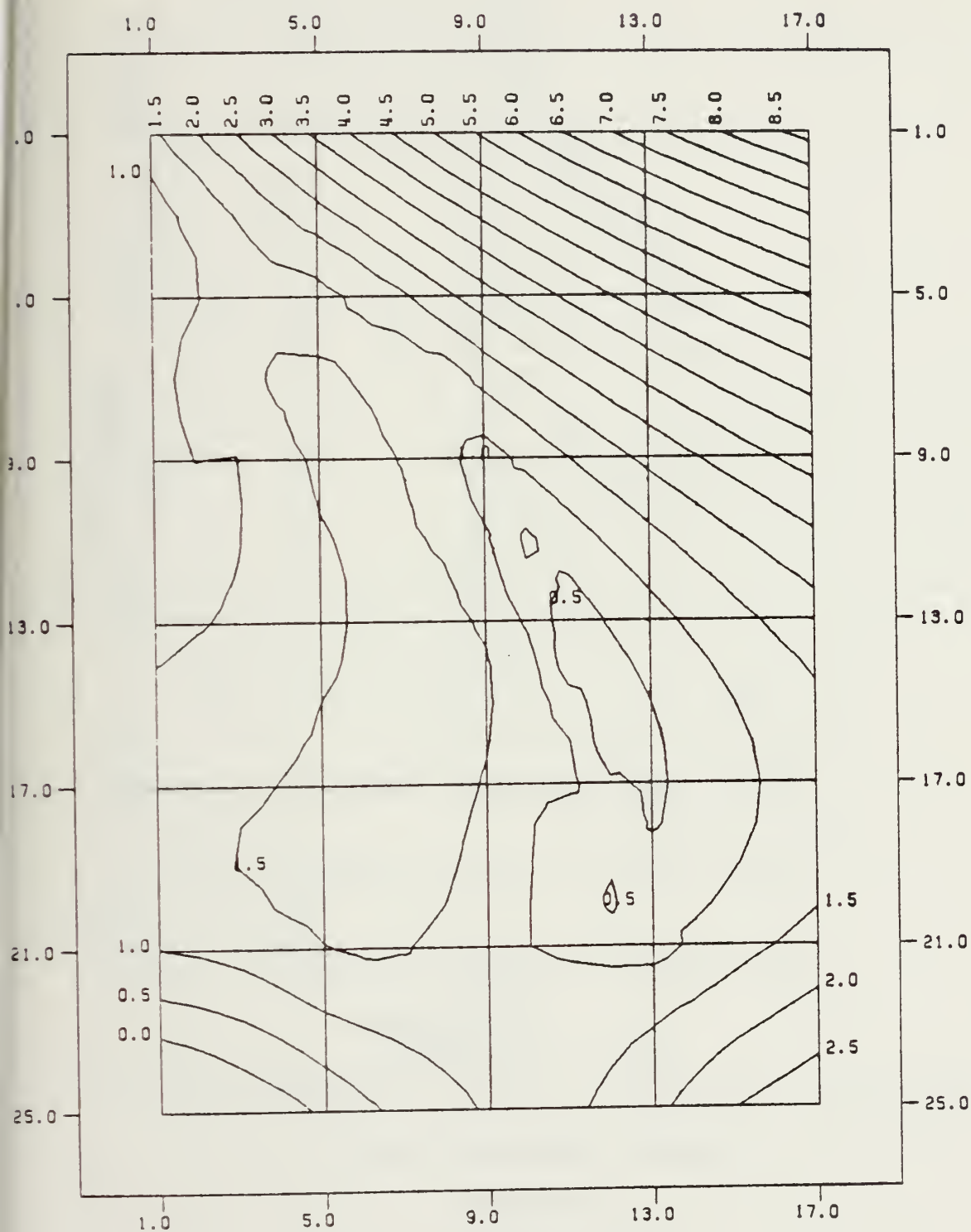
DEPTH (M)  
TRIANGLE BASED QUAD. METHOD  
(Using subroutine PLT3D1)

Figure 38



DEPTH (M)

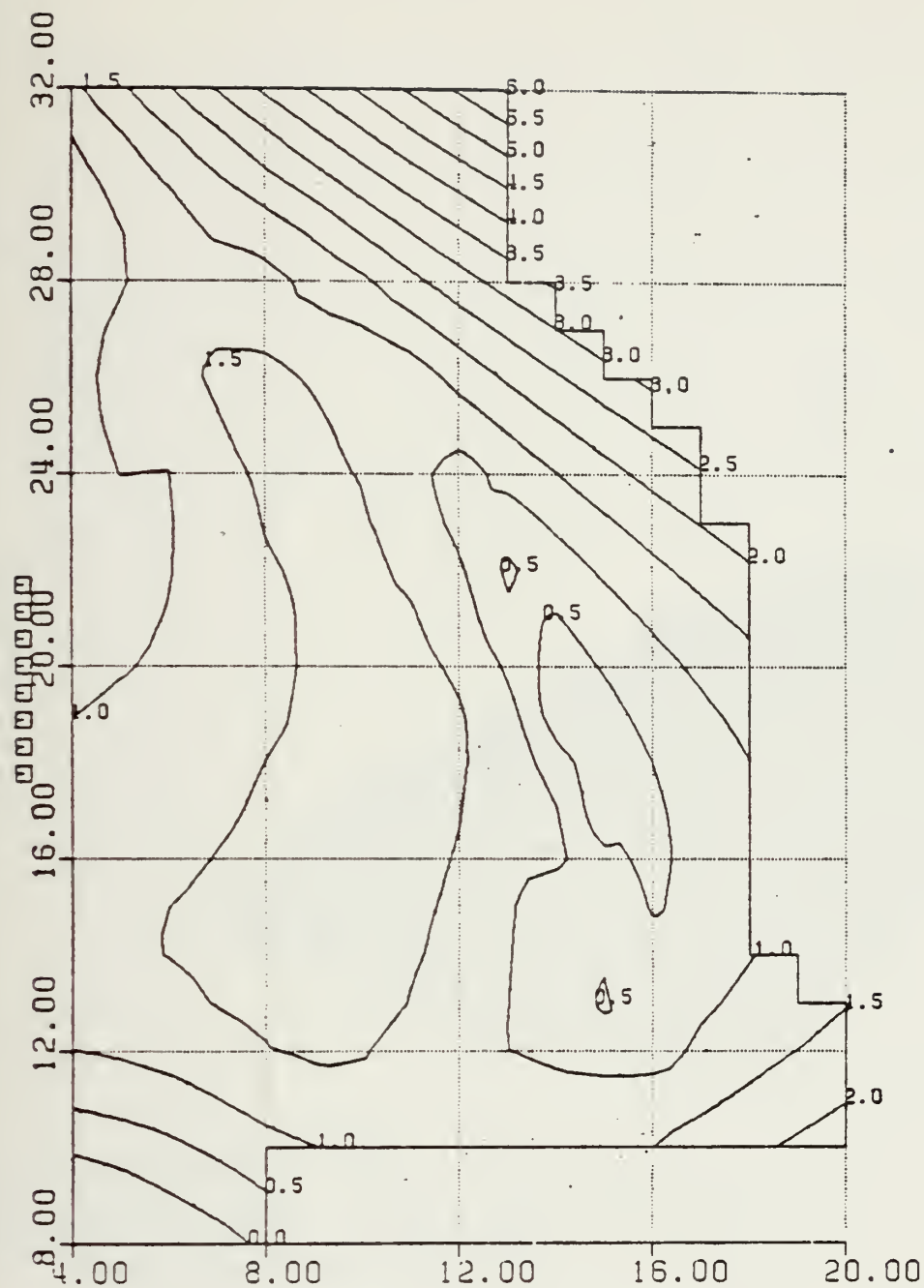
NIELSON'S MIN. NORM NETWORK M.



(Using subroutine CONTUR)

Figure 39





■■■■■■■■■■

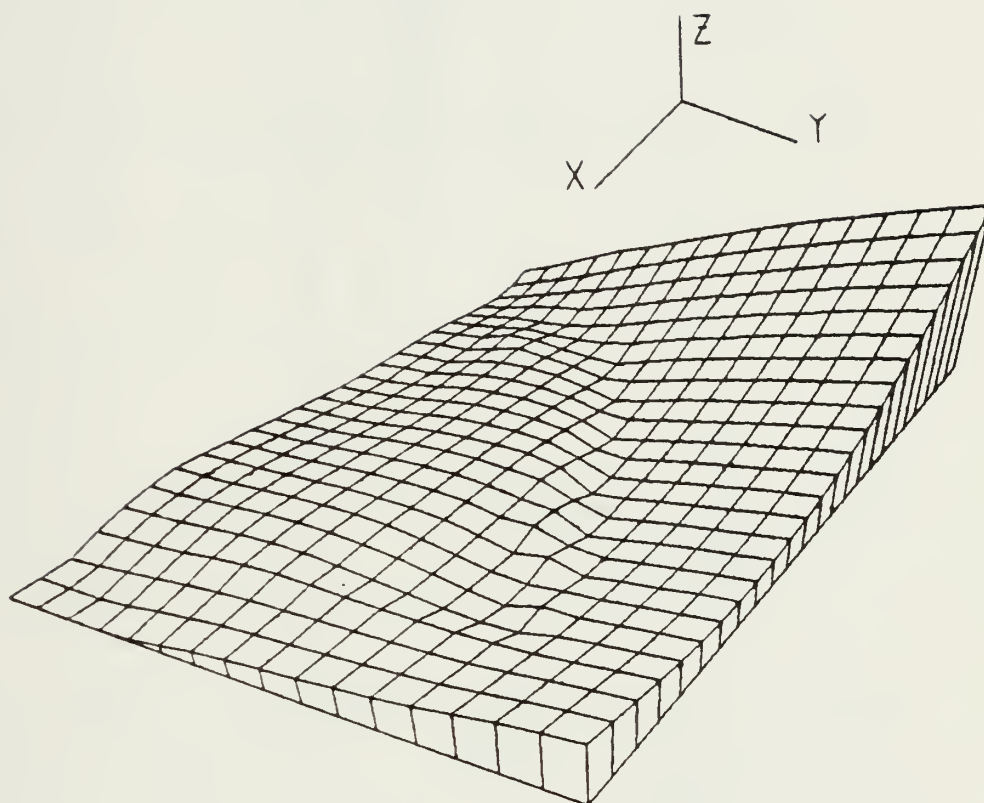
DEPTH (M)

NIELSON'S MIN. NORM NETWORK M.

(Using subroutine CONISD)

Figure 40





DEPTH (M)  
NIELSON'S MIN. NORM NETWORK M.  
(Using subroutine PLT3D1)

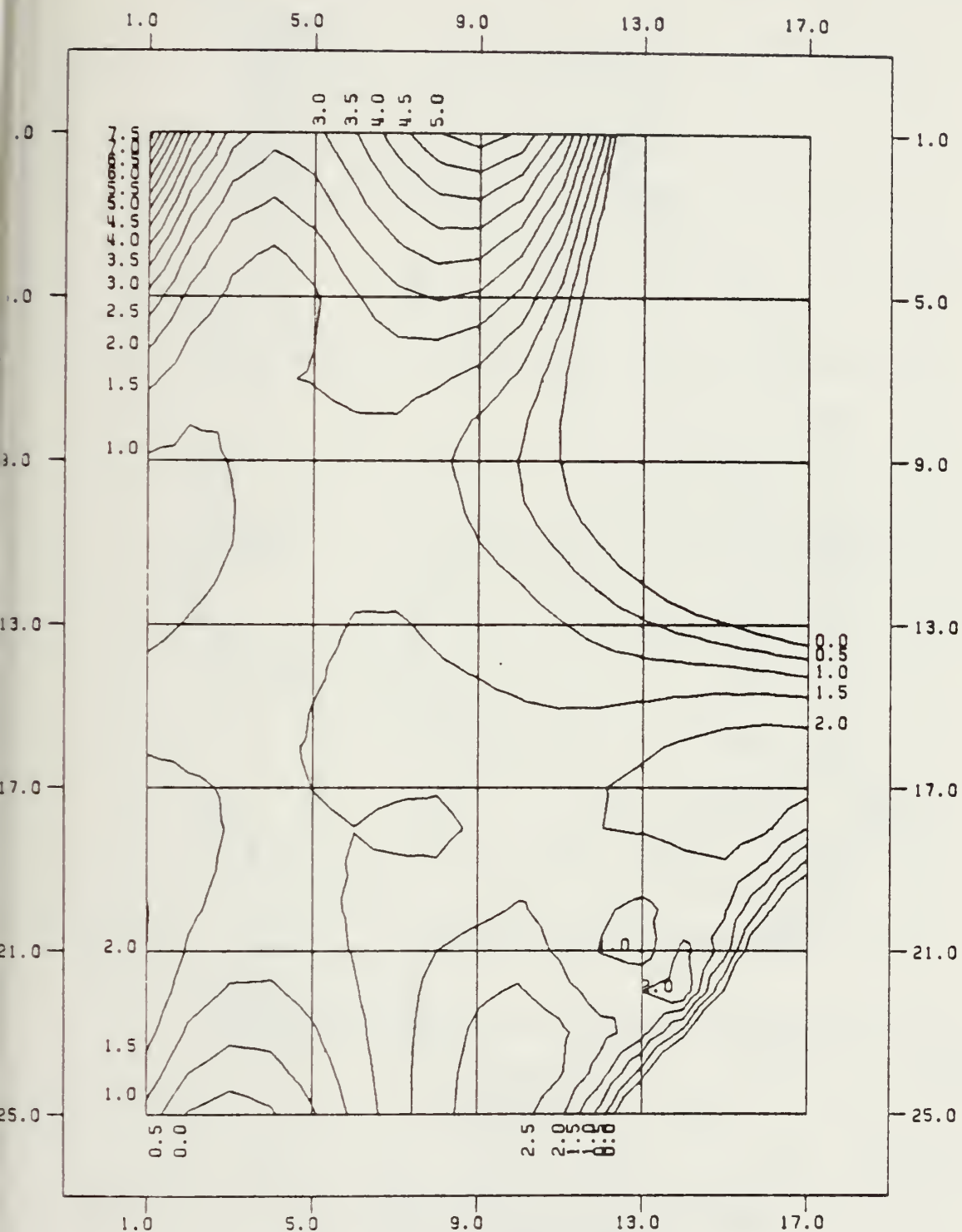
Figure 41





# DEPTH (M)

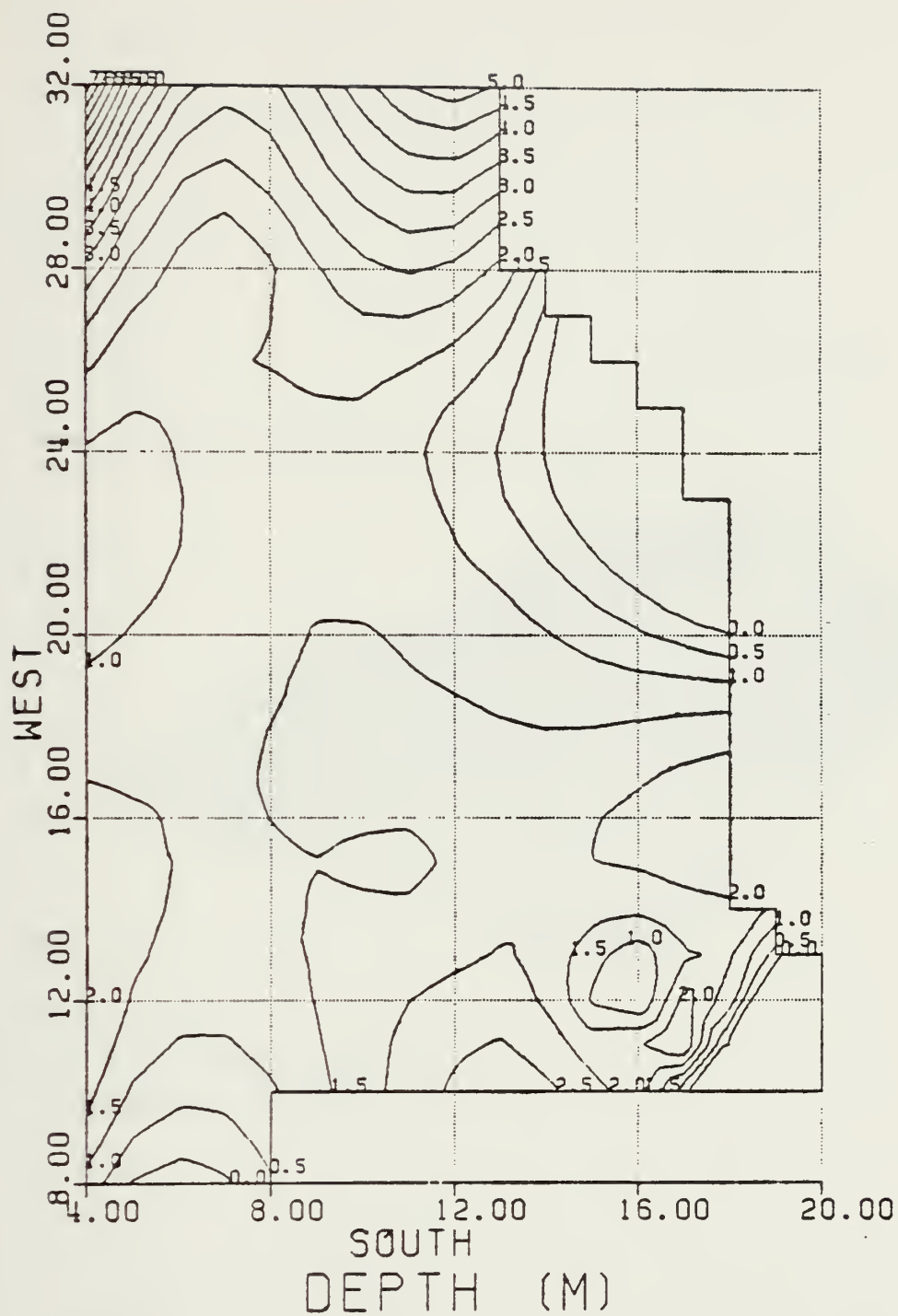
## DELTA SUM BIC. SP. METHOD



(Using subroutine CONTUR)

Figure 42



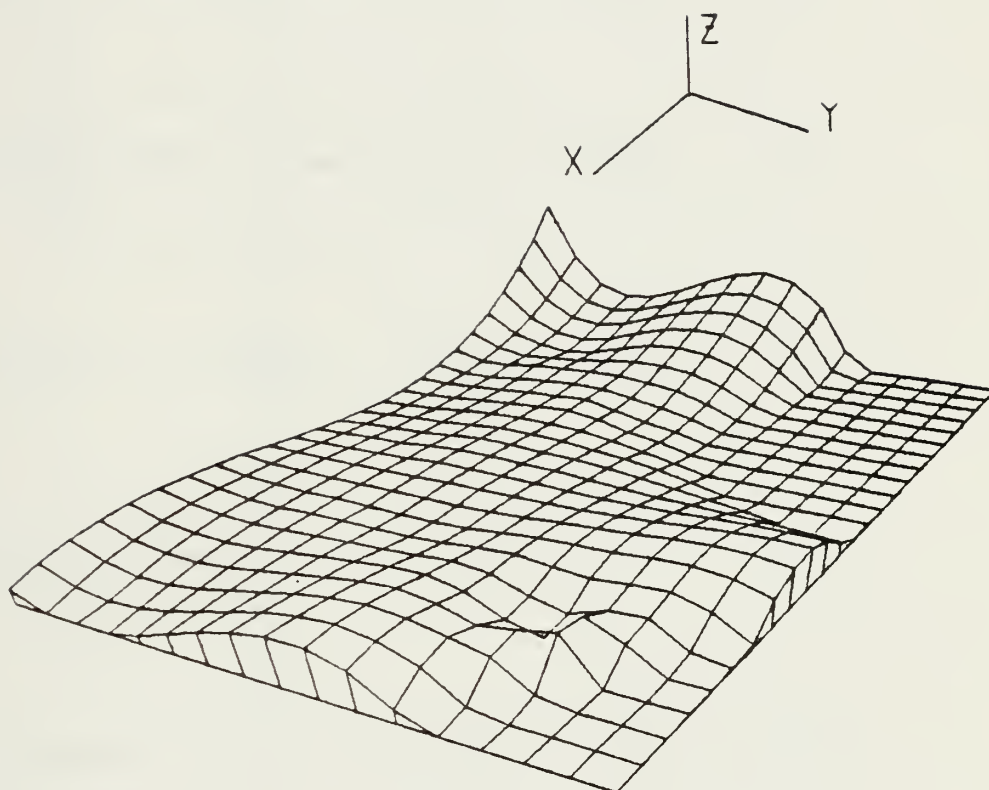


DELTA SUM BIC. SP. METHOD

(Using subroutine CONISD)

Figure 43





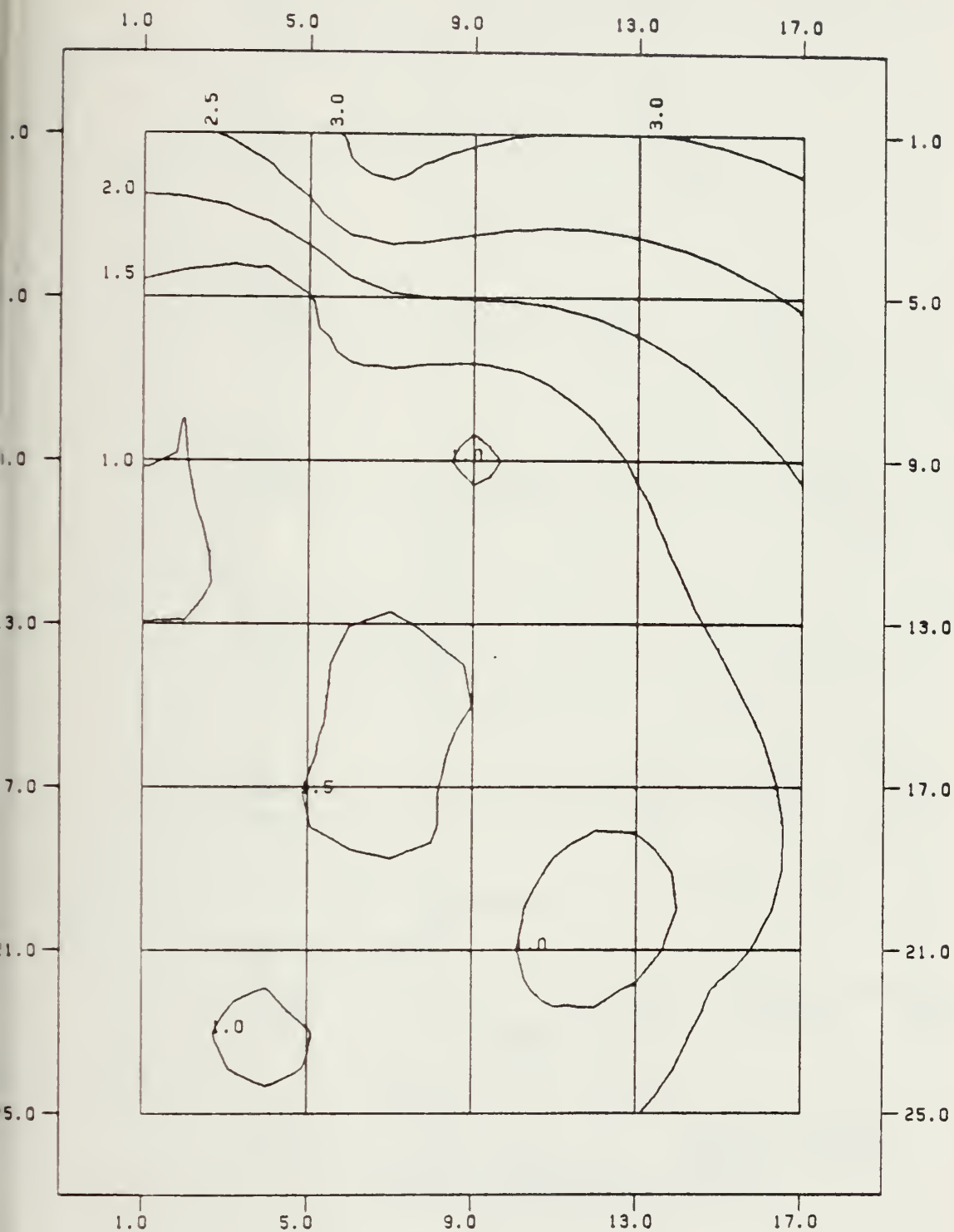
DEPTH (M)  
DELTA SUM BIC. SP. METHOD  
(Using subroutine PLT3D1)

Figure 44



# DEPTH (M)

## MULTIQUADRIC METHOD

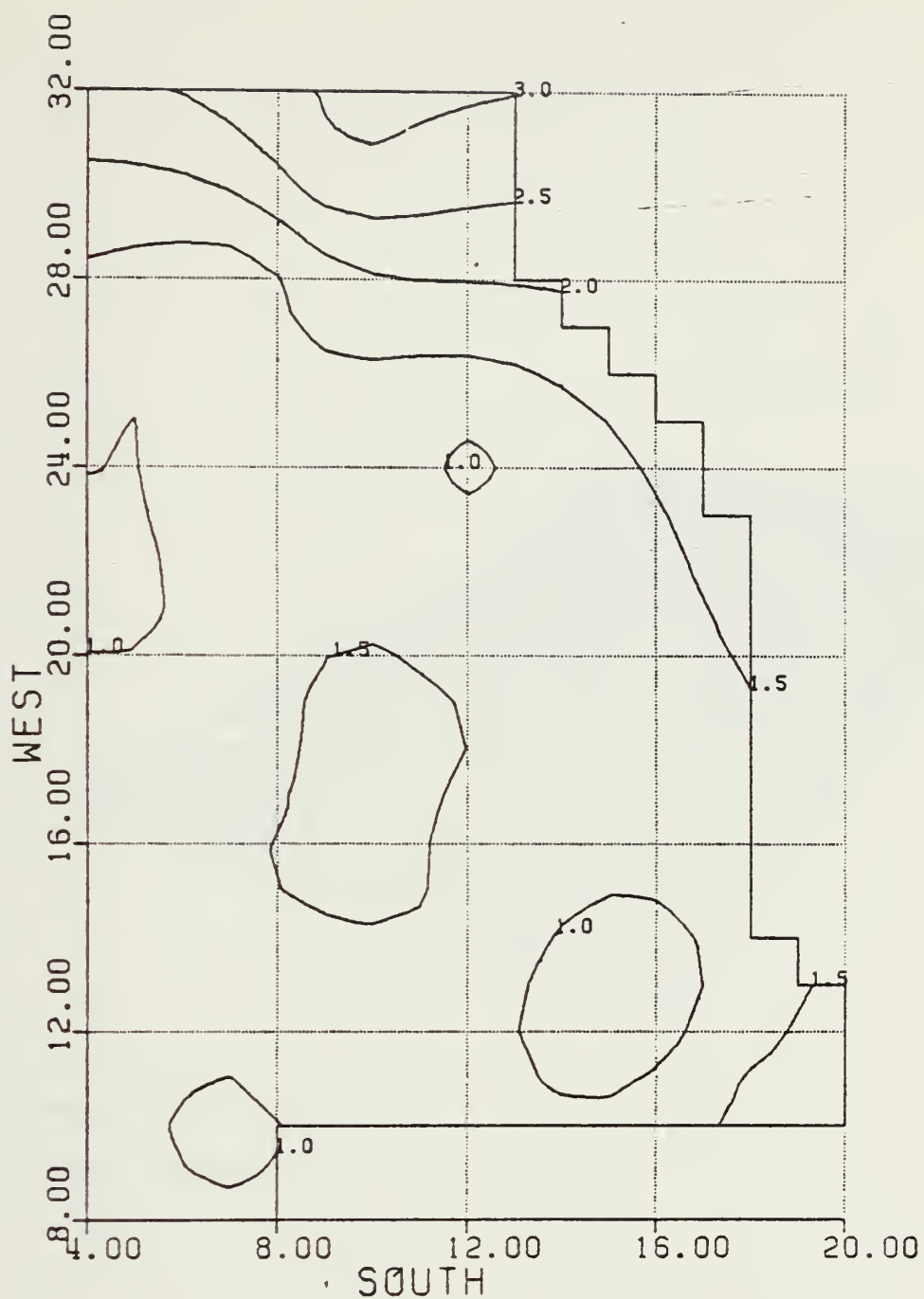


(Using subroutine CONTUR)

Figure 45





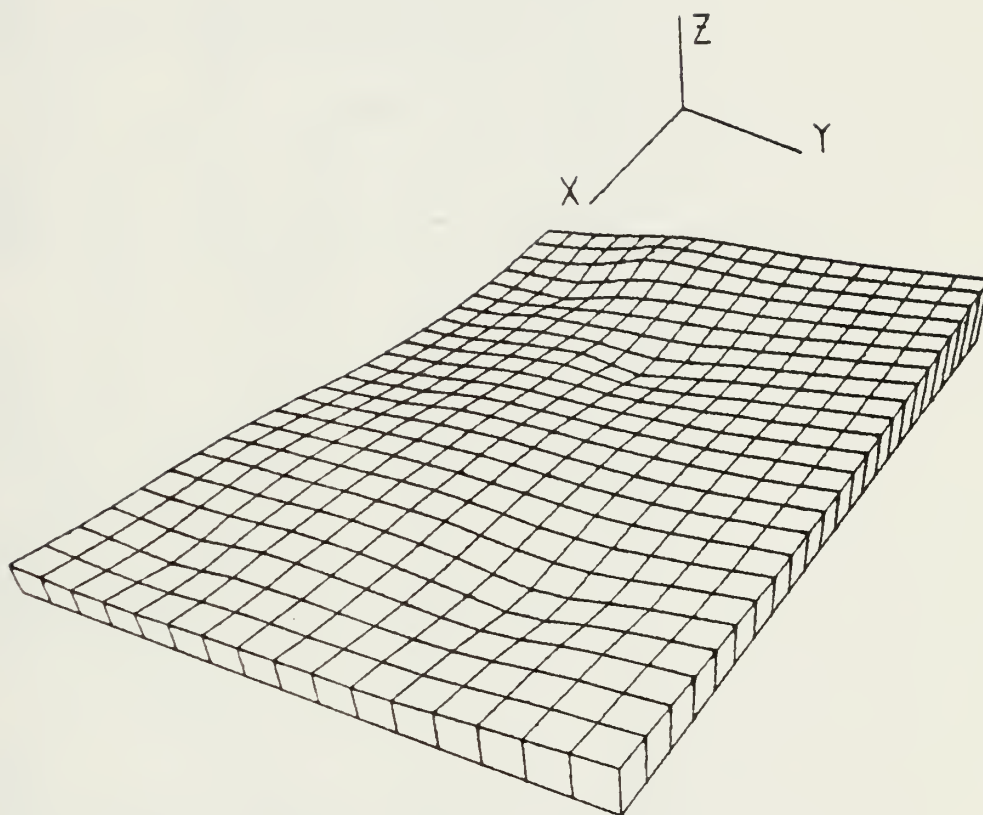


DEPTH (M)  
MULTIQUADRIC METHOD

(Using subroutine CONISD)

Figure 46



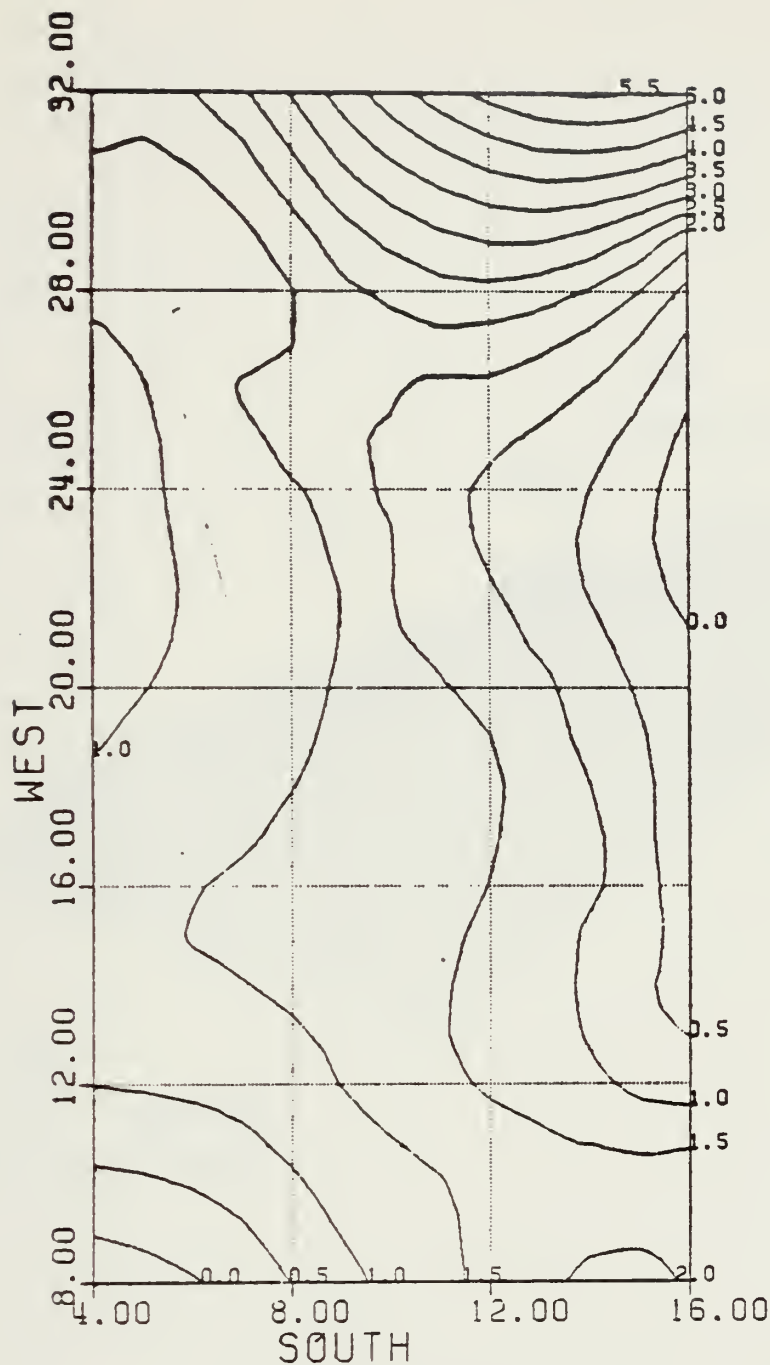


DEPTH (M)  
MULTIQUADRIC METHOD  
(Using subroutine PLT3D1)

Figure 47



APPENDIX G

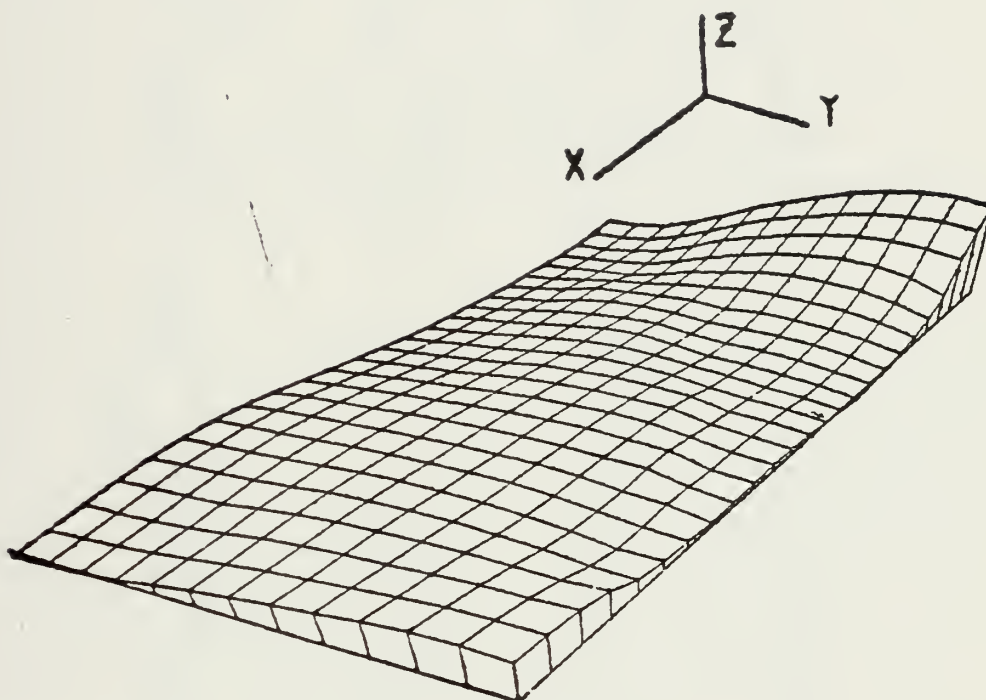


DEPTH (M)  
RALSTON VALLEY. NEVADA

(Using subroutine CONISD)

Figure 48



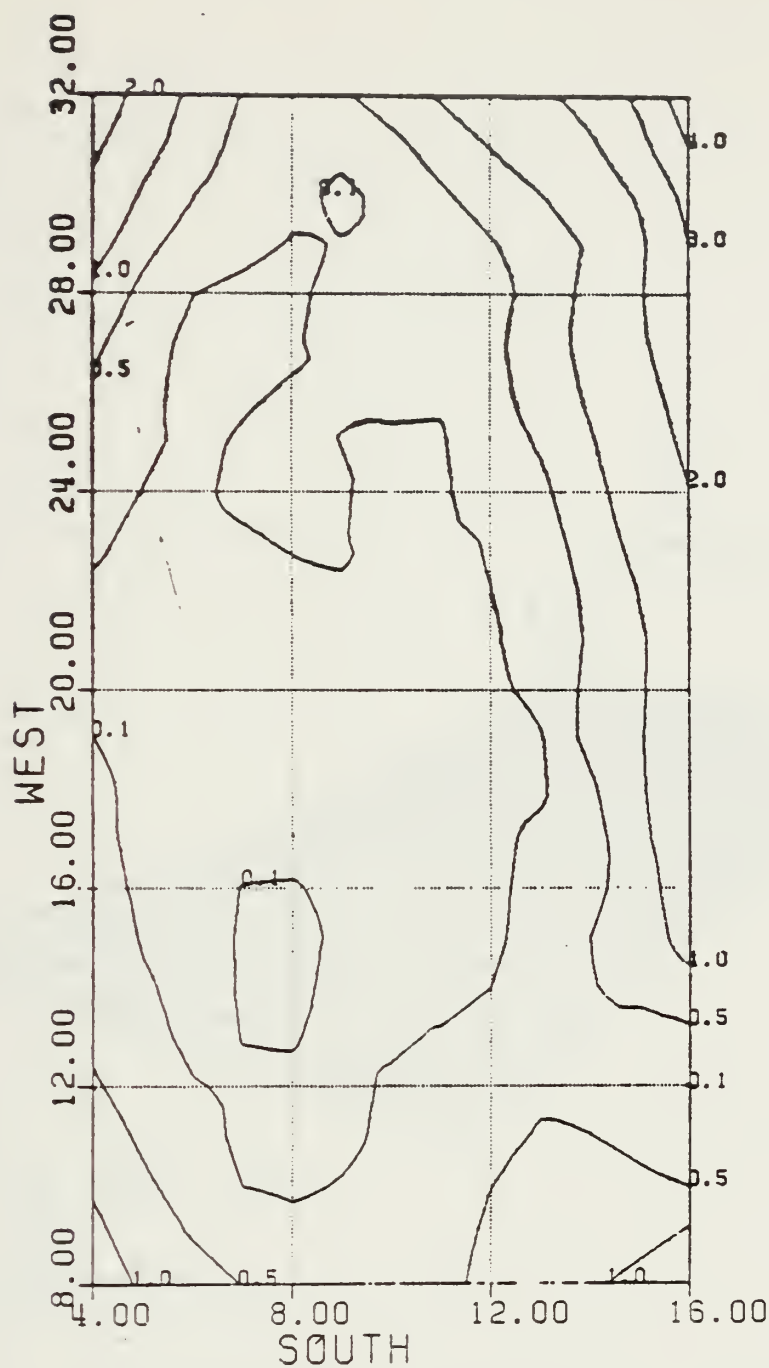


DEPTH (M)  
STON VALLEY, NEVADA  
(Using subroutine PLT3D1)

Figure 49





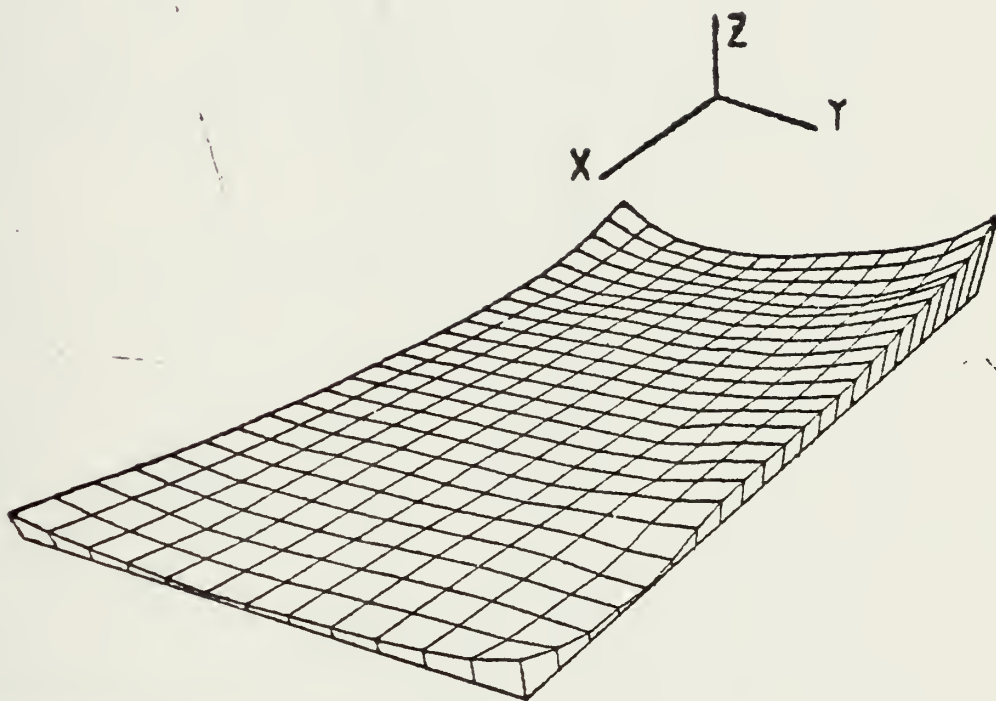


STANDARD DEVIATIONS FOR  
DEPTH (M)

(Using subroutine CONISD)

Figure 50



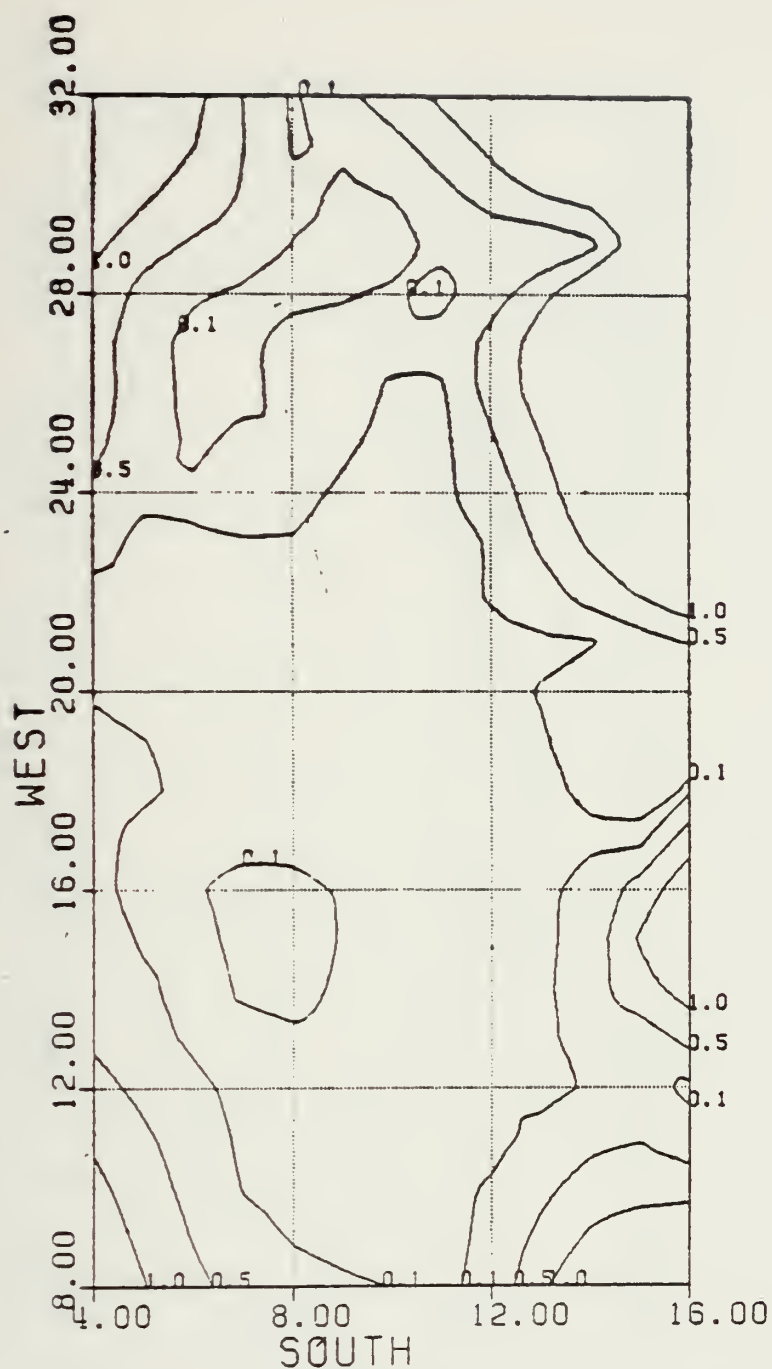


STANDARD DEVIATIONS FOR  
DEPTH (M)  
(Using subroutine PLT3D1)

Figure 51



# APPENDIX H

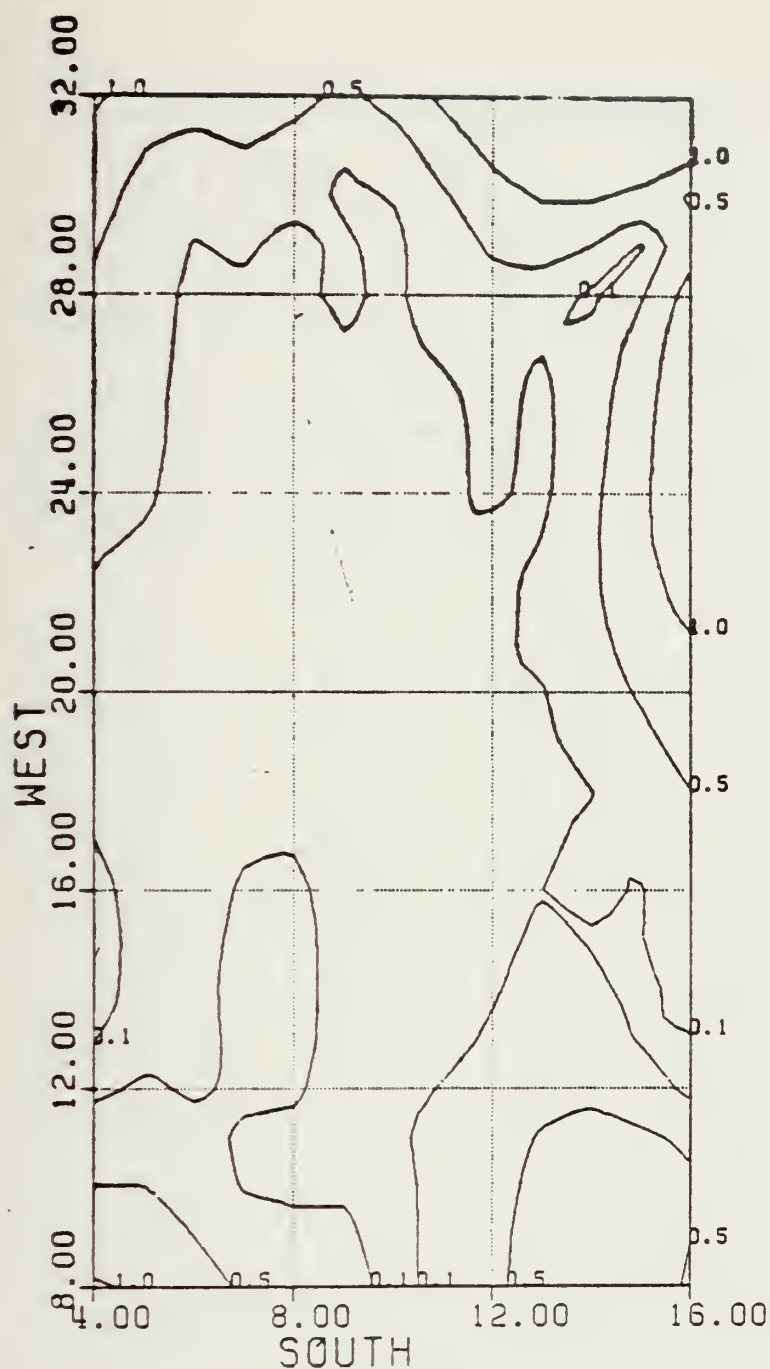


ABS ( QUAD.SHEP.M. - MEAN )  
DEPTH (M)

(Using subroutine CONISD)

Figure 52





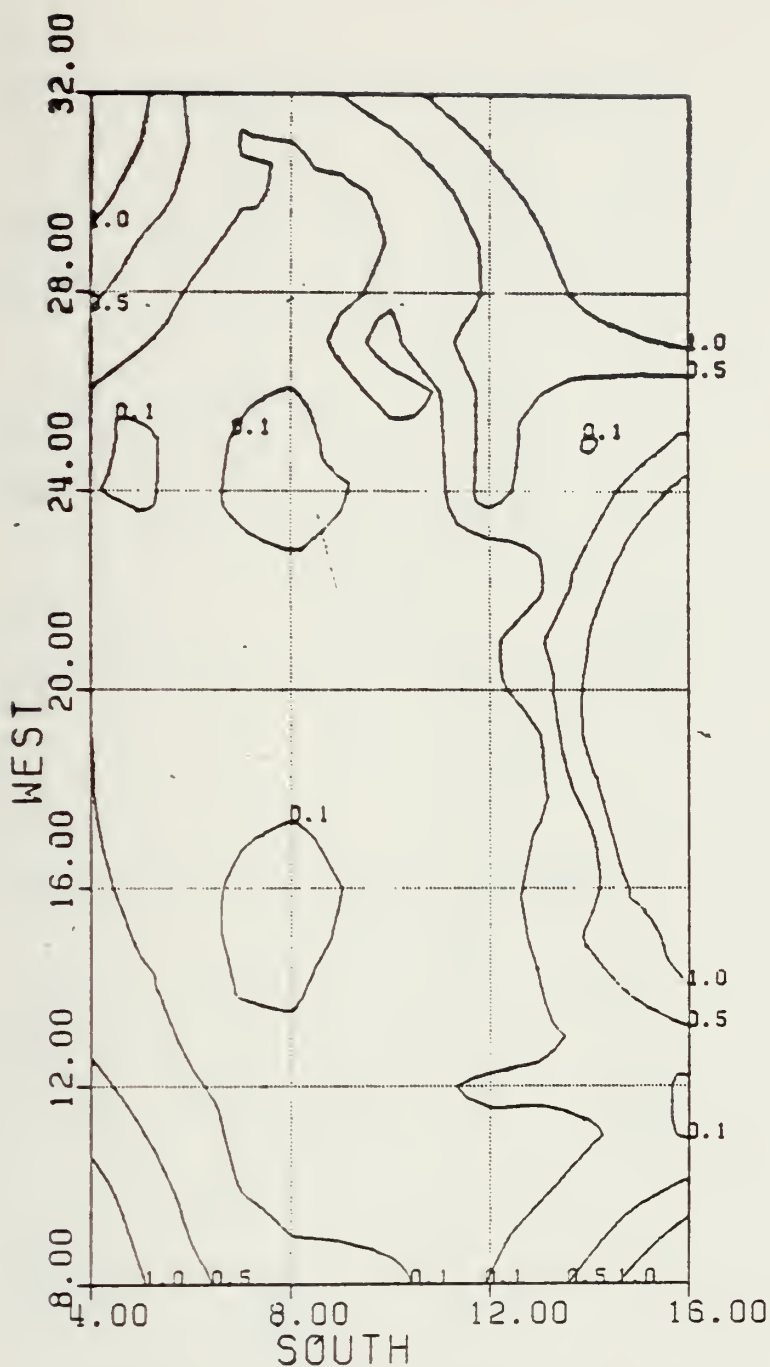
ABS ( LOCAL THIN M. - MEAN )  
 DEPTH (M)

(Using subroutine CONISD)

Figure 53



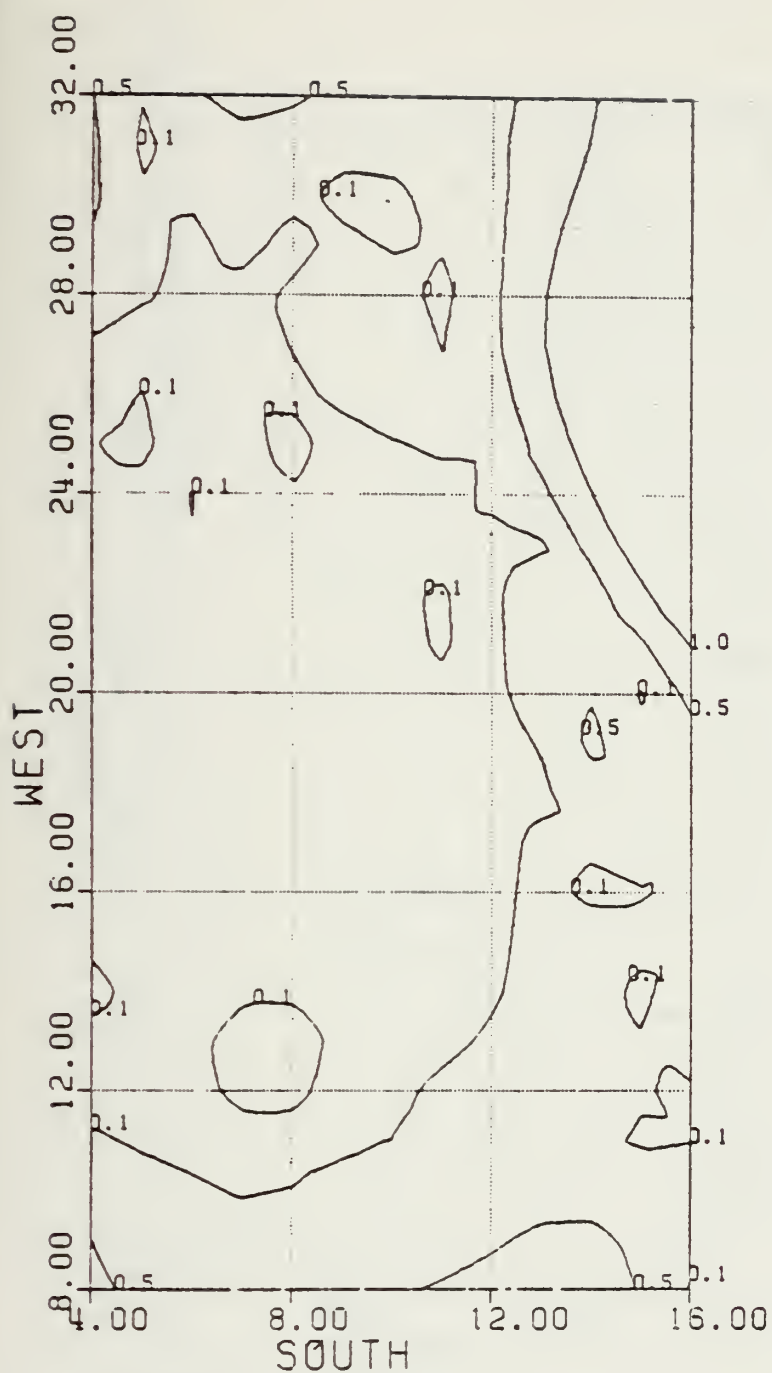




ABS. ( TRI.BASED M. - MEAN )  
 DEPTH (M)  
 (Using subroutine CONISD)

Figure 54





ABS ( NIELSON MIN. M. - MEAN )  
 DEPTH (M)

(Using subroutine CONISD)

Figure 55













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